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Technical Report QP-TR-72-1

## CHAIN SAMPLING

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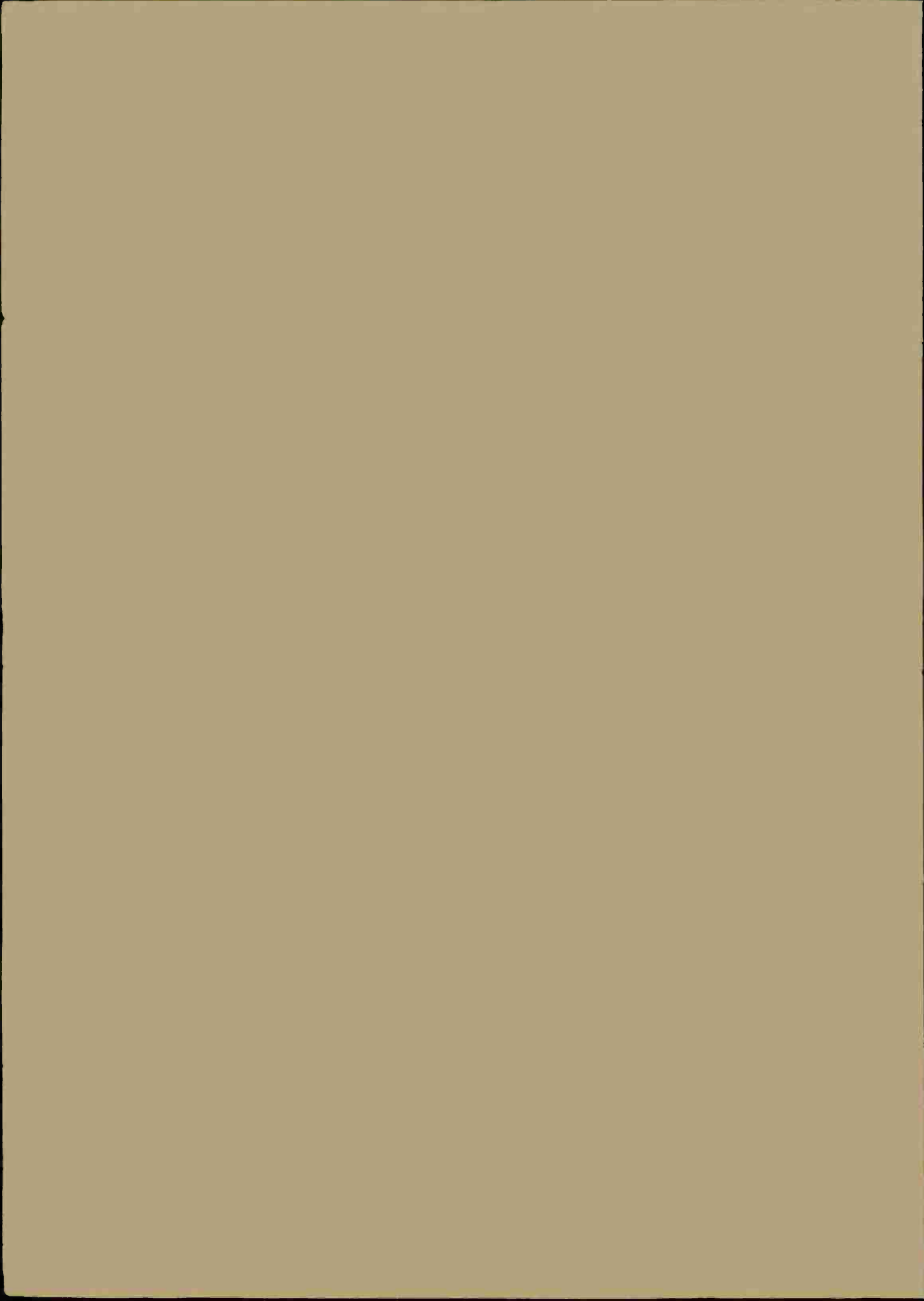
# U.S. ARMY MISSILE COMMAND

*Redstone Arsenal, Alabama 35809*

Advanced Techniques Branch  
Plans and Programs Analysis Division  
Directorate for Product Assurance  
U. S. Army Missile Command  
Redstone Arsenal, Alabama 35809

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## CHAIN SAMPLING

by

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## ABSTRACT

This Technical Report contains a general discussion on acceptance sampling wherein data from previous lots are used as a part of the criteria to determine acceptability of the current lot under consideration. The group of lots from which data are utilized is termed a "chain" of lots (each lot being a link in the chain), and the total process is termed "chain sampling."

The process of chain sampling is described, the theory developed, applications described, and its utility compared to single sampling. In addition, the report presents a somewhat unique method of computerizing the computation of operating characteristic curves for chain sampling plans and offers a limited number of plans for sample sizes 5, 10, 15, and 20 for possible application by its recipients.



## FOREWORD

The application of chain sampling to the general field of acceptance sampling is relatively new and, in fact, is still "low key" with only limited applications being identifiable after considerable contact with government and industry over the past 3 years (see Section 3.0). While the basic theory of chain sampling was set forth by Harold F. Dodge in 1955 and extended in 1964 by H. F. Dodge and K. S. Stephens in a series of technical reports from Rutgers State University under contract to the Office of Naval Research, it appears that little has been done to stimulate application and further analysis of this type acceptance sampling. This report is being written for the primary purpose of attempting to stimulate further research in chain sampling, especially in the applicability area. The authors of this report have drawn heavily on the earlier works of Dodge and Stephens; and although the specific chain sampling plans offered in this document are tailored heavily to Army Missile Command needs, it is hoped that this report will stimulate research and application by others and thus result in additional knowledge being provided for advancement of the state-of-the-art of acceptance sampling.



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## 1.0 INTRODUCTION

Traditionally, acceptance sampling has been utilized as a technique to make a decision to accept or reject a lot of material based on information attained solely from the inspection of a sample from that lot. Chain sampling represents a marked deviation from that concept in that it considers inspection information not only from the lot under consideration but also from previous lots. In this way chain sampling considers historical as well as current inspection results. The method in which the historical results are considered will become obvious to the reader in subsequent paragraphs of this report.

Chain sampling derives its name from the consideration of the fact that acceptance criteria are established for a chain of lots rather than for each individual lot. The entire process is seen as a series of associated operations in which individual samples are like the links in a chain. Acceptance is then based on the sampling results from several links of the chain. This contrasts with single sampling where an accept/reject decision is made solely on data from a specific lot.

The idea of chain sampling was apparently developed by H. F. Dodge and extended by others for application to the inspection of product characteristics when the inspections are costly or destructive. In such cases a plan with a small sample is usually chosen for practical reasons.

Chain sampling is intended to overcome some of the shortcomings of conventional plans having an acceptance number of  $c=0$  without increasing the number of specimens inspected per lot. It attempts to accomplish this by using the cumulative results of several samples and is applicable to situations where production is repetitive under substantially the same conditions.

This technical report provides a brief description of the mechanics of chain sampling; a discussion of its general applicability, offers a variety of plans and their associated operating characteristic curves which may be used as a "handbook" and, finally, provides a fairly comprehensive review of the mathematical principles applicable to chain sampling.

There are many variations in the scheme for doing chain sampling. While appendix A provides a general mathematical treatment of chain sampling, it is most probable that only the mathematically inclined will desire to study it in depth; therefore, it is proper to stress here that the body of this technical report covers only a limited type of chain sampling. Many other variations are possible and in many instances may be more appropriate for a particular application than the particular schemes presented in this report. It is reiterated, however, that the mathematical principles in appendix A cover, or can be modified to cover, all the possible variations.



The Army Missile Command Directorate for Product Assurance has established a rather unique computer program for handling a variety of chain sampling schemes and is available for providing consultation to anyone in the government who is interested in establishing their own capability to generate chain sampling plans tailored to their own particular needs. The basic logic that enables the computer program to operate is described in Appendix B.

## 2.0 MECHANICS OF CHAIN SAMPLING

A chain sample is, as the name implies, a link in a series of links. The chain is a sequence of inspection lots from an essentially continuous process. For illustration purposes, suppose the following criteria are established:

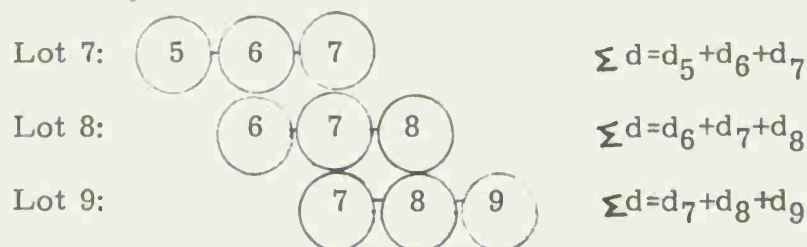
$n$  - the number of units to be inspected from each lot

$K$  - the number of lots in the chain

$c$  - the cumulative acceptance number over  $K$  lots

$d_i$  - the number of defects in the  $i$ th lot

Diagrammatically, the chain sampling plan would operate as follows:



In this plan, if at any time a particular  $\Sigma d$  exceeded  $c$ , the last lot in that chain would be rejected. If in the above example  $d_7 + d_8 + d_9 > c$ , lot 9 would be rejected, lots 8 and 7 would have been previously accepted: lot 8 having been accepted on the basis of the sum of defects from lots 6, 7, and 8; lot 7 having been accepted on the basis of the sum of defects from lots 5, 6, and 7.

It is obvious from the above that  $K$ , the number of lots in the chain, is 3; and that  $c$ , the acceptance number, is not the acceptance number for a single lot but for  $K$  lots (in this example, 3 lots).

The above diagram depicts the "normal operating stage" of chain sampling; that is, one is already in a normal operating chain. One might ask the question, "How did we get in this stage and what do we do if a lot is rejected?" Normally chain sampling will consist of two stages defined as follows:

Initial and restart stage - The stage entered at the very beginning of sampling and the stage entered immediately after any rejection. (This may be envisioned as a clearing sequence upon entering sampling and after any lot rejection.)

Normal operating stage - The stage entered after the initial or restart stage is satisfied (once entered, sampling will remain in this stage until a rejection occurs).

Both of the above stages may involve chains of different lengths and each with its own criteria for acceptance.

With this background a complete chain sampling plan will now be discussed.

Before sampling is begun, certain quantities in addition to sample size,  $n$ , are decided upon.

For the initial and restart stage they are:

1. The number of links (lots) in the initial and restart stage of the plan ( $K_1$ ).

2. The cumulative number of defectives to be allowed in the  $K_1$  samples before a lot will be rejected ( $c_1$ ).

For the normal operating stage they are:

3. The total number of links (lots) in the normal operating stage of the plan ( $K_2$ ) (where  $K_2 \geq K_1$ ).

4. The cumulative number of defectives to be allowed in the  $K_2$  samples, before a lot will be rejected ( $c_2$ ) (where  $c_2 \geq c_1$ ).

Suppose a given plan is ( $K_1=3$ ,  $K_2=4$ ;  $c_1=a_1$ ,  $c_2=a_2$ ). (Refer to Figure 2-1.)

At the start of a program and following a rejected lot, the restart procedure is initiated. In the restart procedure, sample (1) is tested and the number of defectives  $d_1$  in sample (1) are recorded.

Then  $d_1$  is checked against  $c_1$ . If  $d_1$  is equal to, or less than,  $c_1$ , lot (1) is accepted. If  $d_1$  is greater than  $c_1$ , lot (1) is rejected; and since the restart procedure is initiated on rejection of any lot, the above step on the next lot is repeated. This is true for a rejection at any step; therefore, on any subsequent rejection it is to be understood that a return to this step is mandatory.

If lot (1) is accepted, sample (2) is tested and the number of defectives  $d_2$ , in sample (2), is recorded and added to  $d_1$  and the result  $d_1+d_2$  is recorded.

Then  $d_1+d_2$  is compared to  $c_1$ . If  $d_1+d_2$  is equal to, or less than,  $c_1$ , lot (2) is accepted. If  $d_1+d_2$  is greater than  $c_1$ , lot (2) is rejected.

If lot (2) is accepted, sample (3) is tested and the number of defectives  $d_3$  is recorded and added to  $d_1+d_2$  and the result  $d_1+d_2+d_3$  is recorded.

Now  $d_1+d_2+d_3$  is compared to  $c_1$  to see whether the lot (3) will be accepted or rejected.

If  $K_1$  is equal to 3, and rejection of lot three occurs, the restart process begins again with sample (1) and  $c_1$ , all previous samples are forgotten and the results erased.

If  $K_1$  is equal to 3, and acceptance of lot three occurs, the normal operating procedure begins.

Sample (4) is tested and  $d_4$  is recorded and added to  $d_1+d_2+d_3$ , but the result  $d_1+d_2+d_3+d_4$  is checked against  $c_2$ . If  $d_1+d_2+d_3+d_4$  is equal to or less than  $c_2$ , lot 4 is accepted. If  $d_1+d_2+d_3+d_4$  is greater than  $c_2$ , lot 4 is rejected.

The procedure will be repetitive until sample  $K_2+1$  is reached, regardless of the value of  $K_2$ .

Now, sample (5) ( $K_2+1$ ) is tested, the result is recorded and the sum  $d_2+d_3+d_4+d_5$  is formed and checked against  $c_2$ . If  $d_2+d_3+d_4+d_5$  is greater than  $c_2$ , lot (5) is rejected.

This process of retaining the number of defectives in the last  $K_2=4$  samples (for the current sample) continues until a rejection of a lot occurs. The process is the same regardless of the value of  $K_2$ .

When a lot is rejected, the restart process begins again with sample (1) and  $c_1$ . All previous results are erased and all previous lots are forgotten.

There are many variations that can be used in chain sampling. For instance, if  $(K_1=0, c_1=0)$  and  $(K_2=1, c_2=0)$ , the chain process would in essence disappear and the plan would be one in which a single lot will be accepted or rejected depending on whether any defectives were found in the lot. This is a case of a one lot chain which is synonymous with single sampling. Another variation would be to set  $(K_1=0, c_1=0)$  and  $(K_2=3, c_2=2)$ . This would represent a chain plan with no initial or restart stage and a simple three lot chain. In this case, a rejection would merely require that the chain be started anew as if the normal chain were the restart stage.

As in any type of acceptance sampling, the properties of a chain sampling plan are to a large degree indicated by its operating characteristics (OC) curve. In chain sampling if rather large values of  $K_1$  and  $K_2$  are chosen, the computation of OC curves are rather difficult. Appendix A covers thoroughly the mathematical procedures used by the Army Missile Command, and Appendix B covers the logic used in a computer program. Appendix A is fairly complex and is written for the mathematician familiar with the techniques utilized. For the less mathematically inclined, a very simple case will be chosen and a formula for computation

of probability of acceptance ( $P_a$ ) developed. It is to be stressed, however, that this procedure becomes too cumbersome for practical use and is presented only to acquaint the reader with the logic involved in computing OC curves for chain plans.

Suppose a desired plan is established with parameters  $K_1=0$ ,  $c_1=0$ ;  $K_2=3$ ,  $c_2=1$ ;  $n=10$ .

In a three lot chain, the sequences leading to acceptance of a lot can be enumerated as:

- 0 (occurrence of 0 defectives in the sample will always lead to acceptance of the lot)
- R1 (1 defective in a sample preceded by a rejected lot will lead to acceptance of that lot)
- R01 (1 defective in a sample, preceded by 0 defectives and that lot preceded by a rejection)
- 001 (1 defective in a sample preceded by two 0 defective samples)

The probability of lot acceptance,  $P_a$ , is then

$$P_a = P_0 + P_R P_1 + P_R P_0 P_1 + P_0 P_0 P_1$$

$$P_a = P_0 + P_1 [P_R(1 + P_0) + P_0^2]$$

where  $P_0$  = binomial probability of 0 defectives in a sample of 10

$P_1$  = binomial probability of 1 defective in a sample of 10

$P_R$  = probability of lot rejection ( $1 - P_a$ )

Algebraically the  $P_a$  equation can be solved as follows:

$$P_a = P_0 + P_1 P_R (1 + P_0) + P_1 P_0^2$$

$$P_a - P_1 P_R (1 + P_0) = P_0 + P_1 P_0^2$$

$$P_a - P_1 (1 + P_0) + P_a P_1 (1 + P_0) = P_0 (1 + P_1 P_0)$$

$$P_a [1 + P_1 (1 + P_0)] = P_1 (1 + P_0) + P_0 (1 + P_1 P_0)$$

$$P_a = \frac{P_1 (1 + P_0) + P_0 (1 + P_1 P_0)}{1 + P_1 (1 + P_0)}$$

Individual points on the OC curve can be computed from this equation by substituting, for a given fraction defective ( $p'$ ), binomial probability values for  $P_1$  and  $P_0$ .



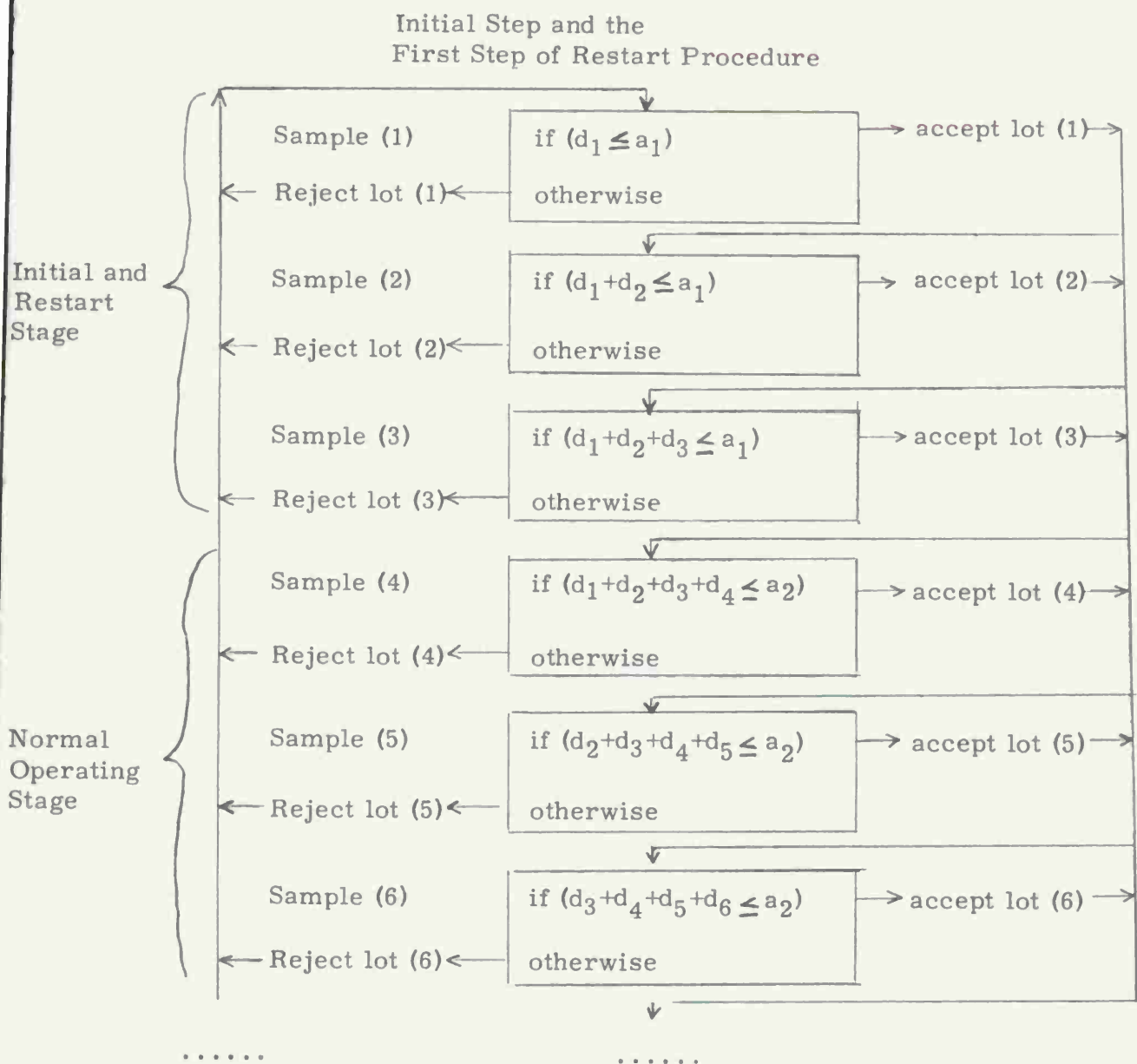


Figure 2-1 Diagram of Two Stage Chain Plan  
( $K_1=3$ ,  $c_1=a_1$ ;  $K_2=4$ ,  $c_2=a_2$ )

### 3.0 APPLICATION OF CHAIN SAMPLING

It has been pointed out that chain sampling is applicable to a sequence of production lots from an essentially continuous process, where each lot is expected to have substantially the same quality as every other lot in the sequence. It is also expected that the lots will be presented for inspection in the order of their production. The procedure is not considered to be suitable for intermittent or job-lot production or to occasional purchases. The main idea behind chain sampling is that, if a process is producing a fairly constant quality product, a slight increase in the number of defectives in the sample may be considered due to chance rather than a change in lot quality.

As previously stated, chain sampling is intended to overcome some of the shortcomings of conventional plans having an acceptance number of  $c=0$  without increasing the number of specimens to be inspected per lot. More generally, it is possible to devise a chain plan whose acceptance probabilities closely match any given single plan with considerable reduction in sample size and inspection cost. (Figure 3-1 provides a comparison of a single sampling plan with a chain plan with a 50% reduction in sample size.) Chain sampling is therefore applicable to situations where inspections are costly and/or destructive so that normally only a small number of samples can be afforded per lot. One application of chain sampling which is employed at MICOM for this reason is the acceptance-test firing of missiles. Obviously in such a situation the economy to the government is considerable in both time and money.

Another possible application of chain sampling takes advantage of lower lot sample sizes to decrease the lot sizes instead of saving money and thereby improve the sensitivity of the plan to change. For example, it is often necessary because of low production rate and high cost of items to define a month's production as a lot. In this case, it is preferable to utilize chain sampling to reduce the lot size to one-half month's production since homogeneity of production is harder to maintain over such a long time period and since the value of material subject to rejection is excessively high.

In the selection of chain sampling plans, caution should be exercised. There are combinations of  $K_1c_1$  with  $K_2c_2$  that are not logical. Two types of such combinations are discussed below.

(1) Very loose  $K_1c_1$  restart stage with very tight  $K_2c_2$  normal stage.

It is not logical to have loose criteria for restart and tight criteria for normal. For example, suppose a plan is selected that has the following parameters  $K_1=2$   $c_1=2$ ,  $K_2=2$   $c_2=1$ ,  $n=5$ . These parameters allow combinations (over two lots) of two failures in ten units in the initial and restart stage and only one failure in ten units in the normal stage. The initial and restart stage is thus much looser than the normal stage. It does not make sense to allow loose criteria during initiation of sampling and immediately after rejecting a lot and go to much more strict criteria while in the normal state. The point to be made is that while this plan might yield a desirable operating characteristic curve, practical analysis should not allow it to be chosen for application.

(2) Extremely tight  $K_1c_1$  criteria coupled with very loose  $K_2c_2$  criteria.

Again, thinking of the  $K_1c_1$  criteria as an initial and restart stage after rejection, it is perfectly logical to establish these criteria tighter than the normal state ( $K_2c_2$ ) criteria; however, here we are alluding to severely tight  $K_1c_1$  criteria as compared to the  $K_2c_2$  criteria. As an example suppose we have the following criteria for a chain plan:  $K_1=2$   $c_1=1$ ,  $K_2=3$   $c_2=5$ ,  $n=10$ . These parameters allow only one defective over 20 units in the initial and restart stage and yet allow five defectives out of 30 in the normal state stage. While this plan might give an acceptable operating characteristic curve, it does not appear very logical for most applications, especially, if one is dealing with sampling for a performance parameter such as reliability which cannot be varied as easily as a simple quality characteristic such as a dimension. In other words, there are definite limits on the "good" side which the contractor cannot be expected to exceed without extensive effort and perhaps extensive redesign. The sampling plans being considered must be analyzed from this practical application viewpoint and the most optimum one chosen.

The above examples represent only a few of the factors which must be considered in selection of a suitable chain sampling plan. The use of this type of acceptance sampling is quite new and we still have a lot to learn concerning proper plan selection. It is hoped that publication of this

report will stimulate interest in the use of chain sampling and will lead to a better understanding of the procedure.



Figure 3-1 Comparison of OC Curves for Single Sampling and Chain Sampling

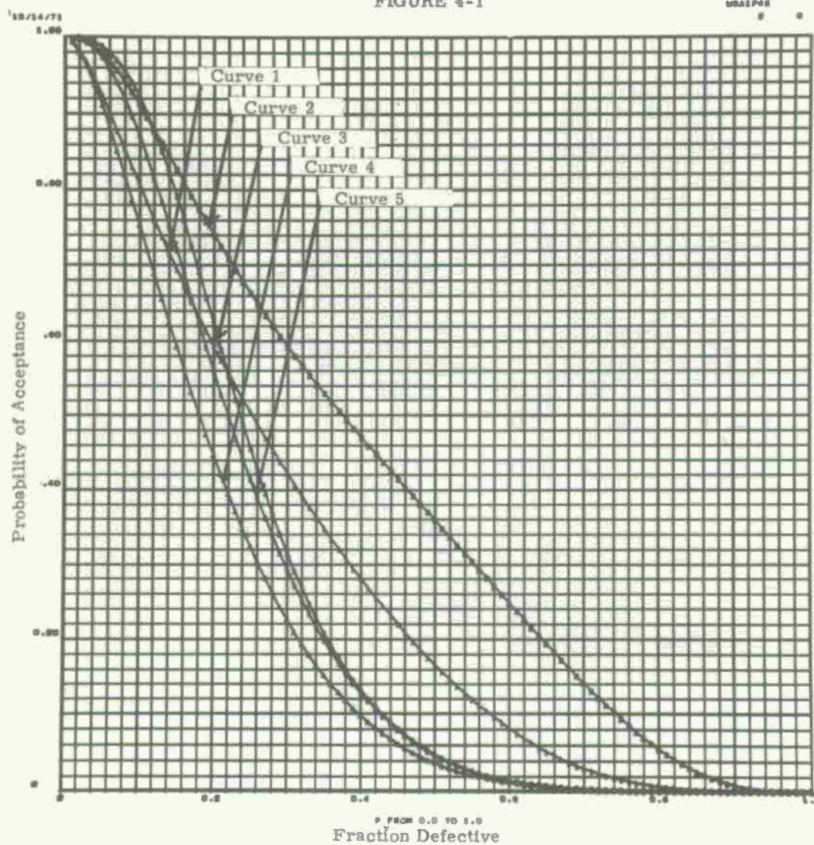
## SECTION 4.0

### CHAIN SAMPLING PLANS - $n=5$





FIGURE 4-1

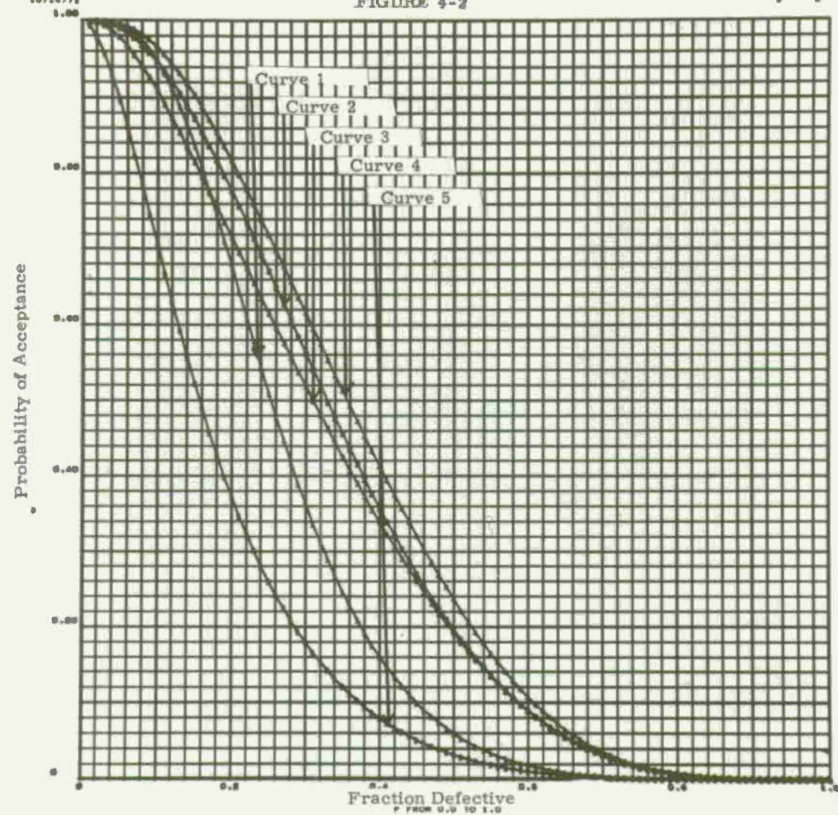


## CHAIN SAMPLING PLAN DATA

SAMPLE SIZE: 5

TABLE 4-1

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=0$ $C_1=0$ $K_2=3$ $C_2=1$	Curve 2 $K_1=0$ $C_1=0$ $K_2=3$ $C_2=2$	Curve 3 $K_1=0$ $C_1=0$ $K_2=2$ $C_2=2$	Curve 4 $K_1=1$ $C_1=0$ $K_2=3$ $C_2=1$	Curve 5 $K_1=1$ $C_1=0$ $K_2=3$ $C_2=2$
.05	.92	.98	.99	.91	.97
.10	.81	.91	.92	.75	.87
.15	.69	.83	.79	.59	.72
.20	.59	.74	.62	.45	.56
.25	.50	.66	.46	.32	.41
.30	.42	.59	.31	.23	.29
.35	.35	.53	.21	.15	.20
.40	.28	.47	.13	.10	.13
.45	.22	.41	.08	.06	.08
.50	.17	.36	.05	.04	.05
.55	.12	.30	.03	.02	.03
.60	.08	.25	.01	.01	.01
.65	.05	.19	.006	.006	.006
.70	.03	.14			
.75	.02	.09			
.80	.007	.05			
.85	.002	.03			
.90		.008			

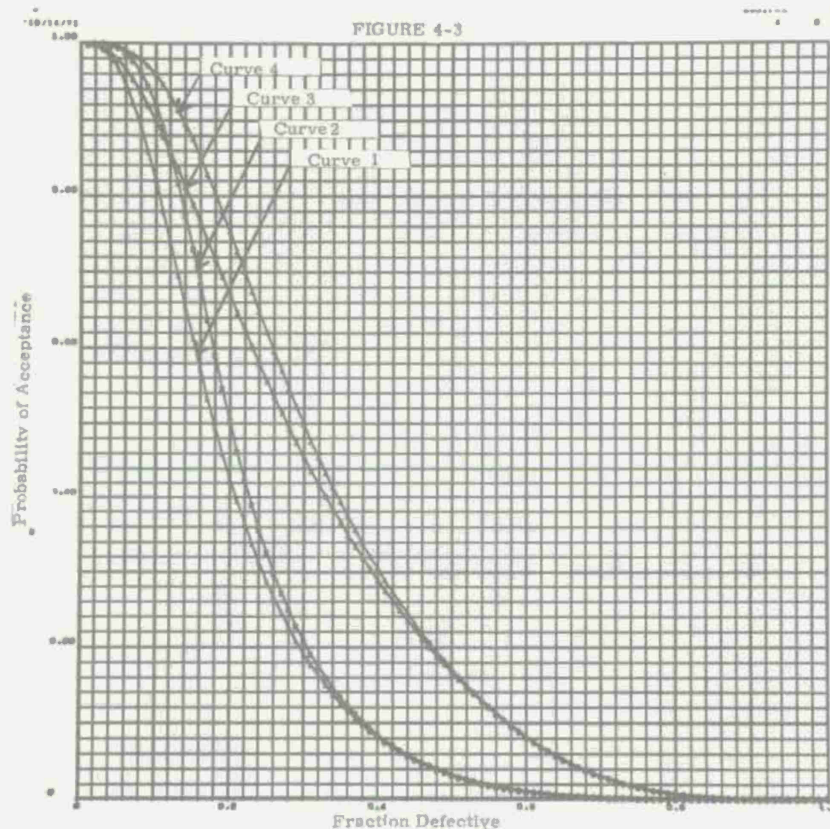


## CHAIN SAMPLING PLAN DATA

SAMPLE SIZE: 5

TABLE 4-2

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=1$ $C_1=0$ $K_2=3$ $C_2=3$	Curve 2 $K_1=1$ $C_1=1$ $K_2=2$ $C_2=2$	Curve 3 $K_1=1$ $C_1=1$ $K_2=3$ $C_2=2$	Curve 4 $K_1=1$ $C_1=1$ $K_2=3$ $C_2=3$	Curve 5 $K_1=2$ $C_1=0$ $K_2=3$ $C_2=1$
.05	.995	.99	.98	.996	.90
.10	.95	.95	.91	.97	.70
.15	.84	.87	.82	.90	.52
.20	.68	.78	.71	.82	.37
.25	.51	.67	.61	.72	.26
.30	.36	.56	.52	.61	.18
.35	.25	.46	.43	.51	.12
.40	.16	.36	.34	.41	.08
.45	.10	.27	.26	.32	.05
.50	.06	.20	.19	.24	.03
.55	.03	.14	.13	.16	.02
.60	.02	.09	.09	.11	.01
.65	.01	.05	.05	.06	.005
.70		.03	.03	.04	.002
.75		.02	.02	.02	
.80		.01	.01	.01	



# CHAIN SAMPLING PLAN DATA

SAMPLE SIZE: 5

TABLE 4-3

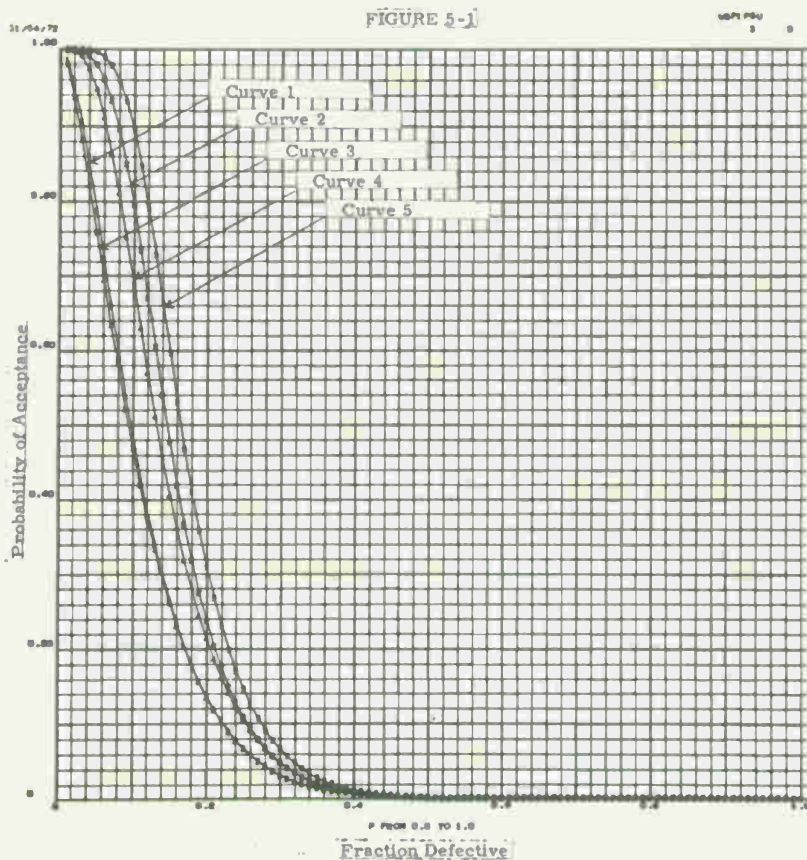
Fraction Defective P	Probability of Acceptance			
	Curve 1 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=2$	Curve 2 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=3$	Curve 3 $K_1=2 \ C_1=1$ $K_2=3 \ C_2=2$	Curve 4 $K_1=2 \ C_1=1$ $K_2=3 \ C_2=3$
.05	.97	.99	.98	.996
.10	.82	.92	.90	.96
.15	.61	.73	.79	.87
.20	.43	.50	.67	.75
.25	.29	.33	.56	.62
.30	.19	.21	.45	.50
.35	.13	.13	.36	.39
.40	.08	.08	.29	.30
.45	.05	.05	.22	.23
.50	.03	.03	.17	.17
.55	.02	.02	.12	.12
.60	.01	.01	.08	.08
.65			.05	.05
.70			.03	.03
.75			.02	.02
.80			.01	.01



## SECTION 5.0

### CHAIN SAMPLING PLANS - $n=10$



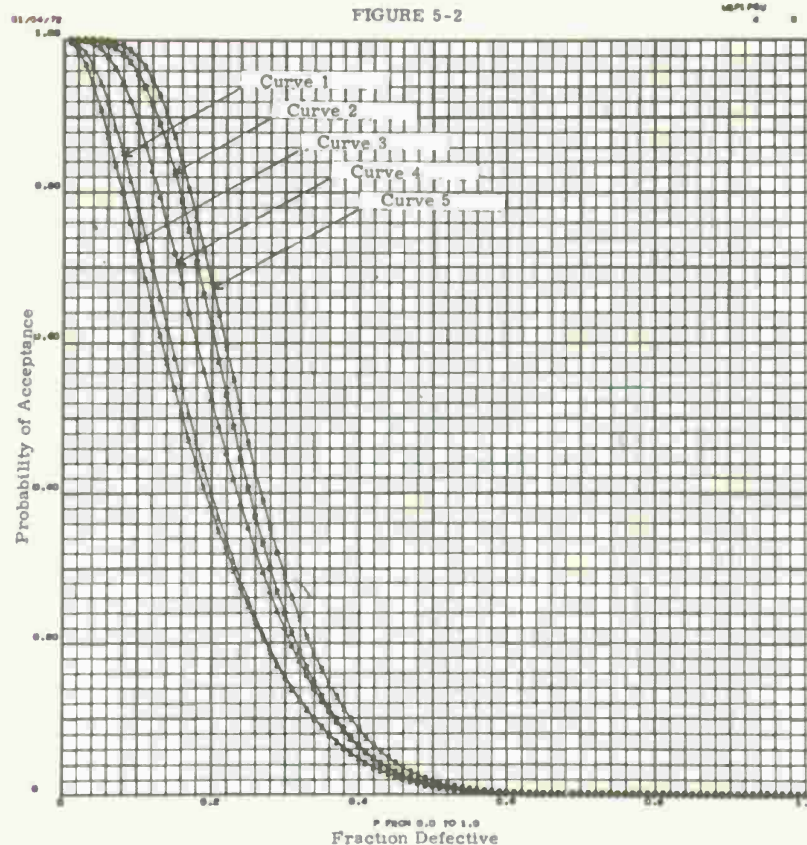


CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 10

TABLE 5-1

Fraction Defective  P	Probability of Acceptance				
	Curve 1 $K_1=1 \ C_1=0$	Curve 2 $K_1=1 \ C_1=0$	Curve 3 $K_1=1 \ C_1=0$	Curve 4 $K_1=1 \ C_1=0$	Curve 5 $K_1=1 \ C_1=0$
	$K_2=2 \ C_2=1$	$K_2=2 \ C_2=3$	$K_2=3 \ C_2=1$	$K_2=3 \ C_2=3$	$K_2=3 \ C_2=5$
0.01	0.99	1.0	0.98	1.0	1.0
0.02	0.95	0.999	0.94	0.998	1.0
0.03	0.91	0.997	0.88	0.99	1.0
0.04	0.85	0.99	0.82	0.97	0.999
0.05	0.79	0.98	0.76	0.95	0.996
0.06	0.72	0.96	0.69	0.91	0.99
0.07	0.66	0.93	0.63	0.86	0.98
0.08	0.598	0.895	0.58	0.81	0.96
0.09	0.54	0.85	0.52	0.75	0.93
0.10	0.48	0.79	0.47	0.69	0.89
0.11	0.43	0.73	0.42	0.63	0.85
0.12	0.38	0.67	0.37	0.57	0.79
0.14	0.30	0.54	0.295	0.46	0.66
0.16	0.23	0.42	0.23	0.36	0.53
0.18	0.18	0.32	0.18	0.28	0.41
0.20	0.14	0.24	0.14	0.21	0.31
0.22	0.10	0.17	0.10	0.16	0.23
0.25	0.067	0.11	0.067	0.10	0.15
0.27	0.0498	0.079	0.0498	0.077	0.11
0.30	0.032	0.048	0.032	0.048	0.066
0.33	0.0199	0.029	0.0199	0.029	0.040
0.37	0.010	0.015	0.010	0.014	0.020
0.41	0.0053	0.00696	0.0053	0.00696	0.0098



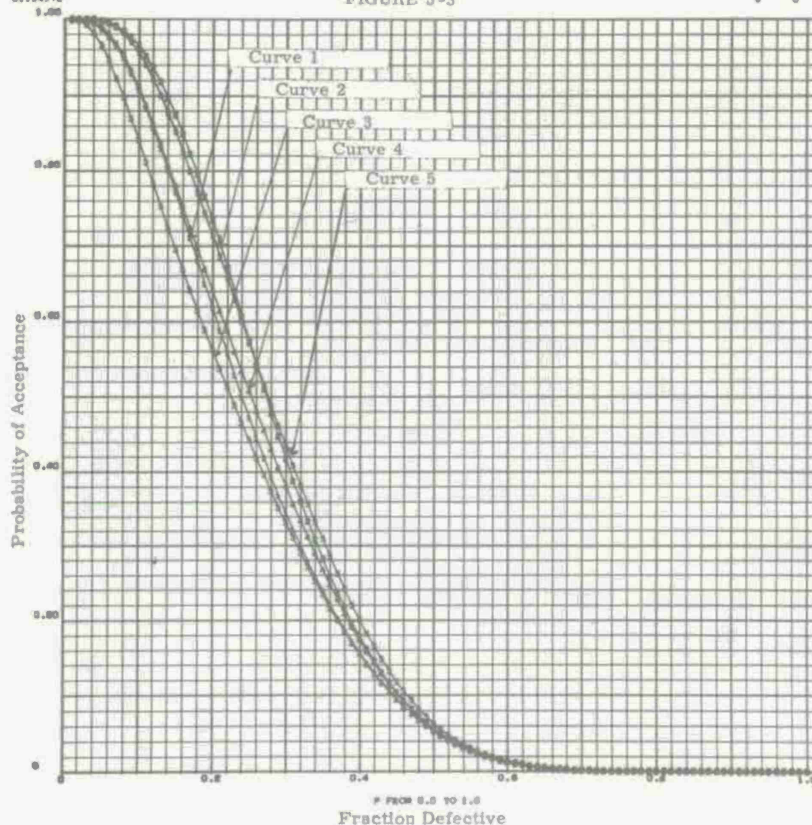


CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 10

TABLE 5-2

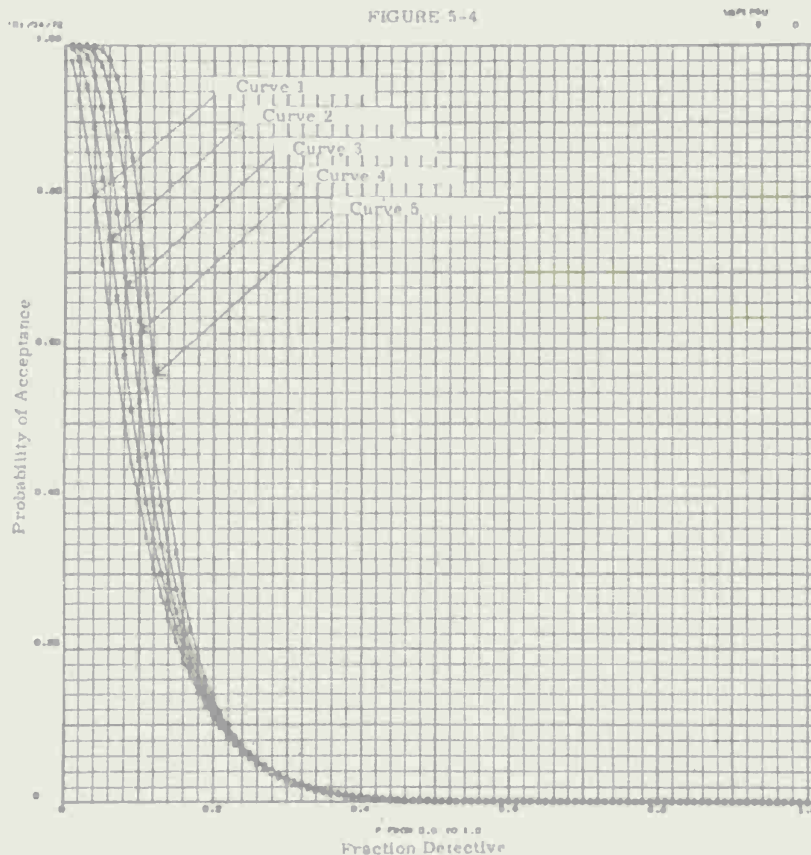
Fraction Defective P	Curve 1 $K_1=1 C_1=1$ $K_2=2 C_2=2$	Curve 2 $K_1=1 C_1=1$ $K_2=2 C_2=4$	Curve 3 $K_1=1 C_1=1$ $K_2=3 C_2=2$	Curve 4 $K_1=1 C_1=1$ $K_2=3 C_2=4$	Curve 5 $K_1=1 C_1=1$ $K_2=3 C_2=6$
0.01	0.999	1.0	0.998	1.0	1.0
0.02	0.99	1.0	0.99	1.0	1.0
0.03	0.98	1.0	0.97	0.999	1.0
0.04	0.97	0.999	0.94	0.995	1.0
0.05	0.94	0.998	0.91	0.99	0.999
0.06	0.92	0.995	0.87	0.98	0.999
0.07	0.89	0.99	0.84	0.96	0.996
0.08	0.85	0.98	0.797	0.94	0.99
0.09	0.82	0.97	0.76	0.92	0.99
0.10	0.78	0.96	0.72	0.89	0.98
0.11	0.74	0.94	0.68	0.86	0.97
0.12	0.699	0.92	0.65	0.83	0.95
0.13	0.66	0.89	0.61	0.79	0.93
0.14	0.62	0.86	0.57	0.76	0.90
0.17	0.50	0.75	0.47	0.64	0.80
0.22	0.34	0.53	0.32	0.45	0.59
0.27	0.21	0.33	0.21	0.29	0.39
0.31	0.14	0.21	0.14	0.19	0.26
0.33	0.11	0.17	0.11	0.16	0.21
0.36	0.078	0.11	0.078	0.11	0.14
0.39	0.054	0.074	0.054	0.072	0.098
0.43	0.031	0.041	0.031	0.040	0.055
0.46	0.020	0.025	0.020	0.025	0.035
0.48	0.015	0.018	0.015	0.018	0.025
0.54	0.0054	0.0061	0.0054	0.0061	0.0083





CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 10  
TABLE 5-3

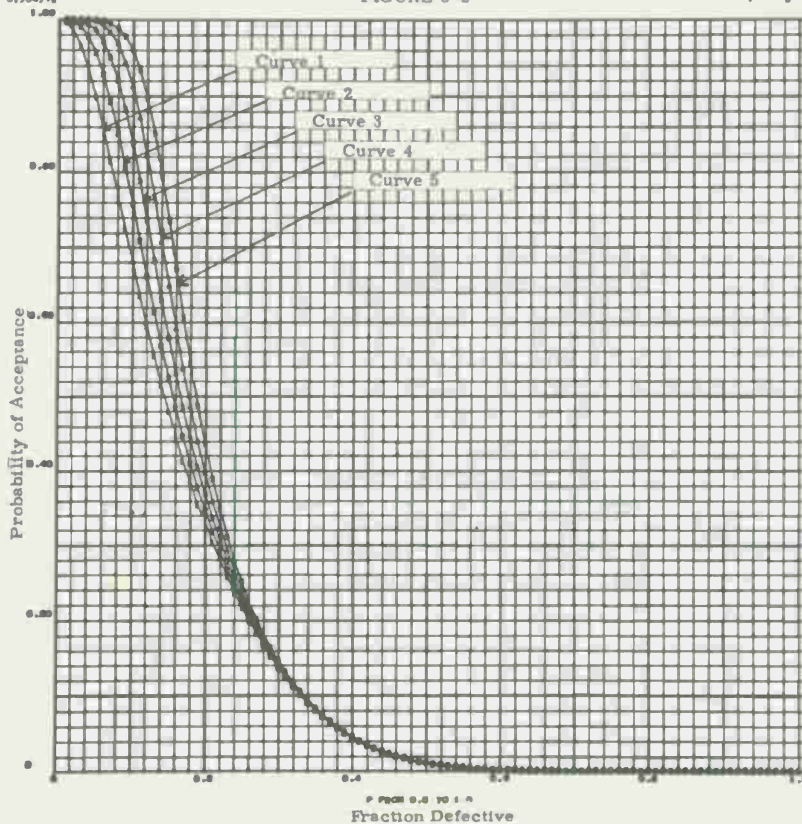
Fraction Defective P	Curve 1 $K_1=1 \ C_1=2$ $K_2=2 \ C_2=3$	Curve 2 $K_1=1 \ C_1=2$ $K_2=2 \ C_2=4$	Curve 3 $K_1=1 \ C_1=2$ $K_2=3 \ C_2=3$	Curve 4 $K_1=1 \ C_1=2$ $K_2=3 \ C_2=4$	Curve 5 $K_1=1 \ C_1=2$ $K_2=3 \ C_2=5$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.999	1.0	0.998	1.0	1.0
0.03	0.998	1.0	0.99	0.999	1.0
0.04	0.99	0.999	0.98	0.996	0.999
0.05	0.99	0.998	0.97	0.99	0.998
0.06	0.98	0.995	0.95	0.98	0.99
0.07	0.96	0.99	0.92	0.97	0.99
0.08	0.95	0.98	0.896	0.95	0.98
0.09	0.93	0.98	0.87	0.93	0.97
0.10	0.91	0.97	0.84	0.91	0.96
0.11	0.88	0.95	0.81	0.89	0.94
0.12	0.86	0.94	0.78	0.86	0.92
0.13	0.83	0.92	0.75	0.83	0.8996
0.14	0.80	0.897	0.72	0.81	0.88
0.15	0.77	0.87	0.696	0.78	0.85
0.16	0.74	0.85	0.67	0.75	0.83
0.17	0.71	0.82	0.64	0.72	0.799
0.21	0.59	0.70	0.54	0.61	0.69
0.24	0.50	0.61	0.46	0.53	0.5995
0.27	0.42	0.51	0.396	0.45	0.52
0.30	0.34	0.42	0.33	0.38	0.43
0.34	0.26	0.31	0.25	0.29	0.33
0.39	0.17	0.196	0.17	0.19	0.22
0.41	0.14	0.16	0.14	0.16	0.18
0.43	0.12	0.13	0.12	0.13	0.15
0.45	0.095	0.10	0.094	0.10	0.12
0.47	0.076	0.082	0.076	0.081	0.092
0.49	0.0599	0.064	0.0598	0.064	0.071
0.53	0.036	0.037	0.036	0.037	0.041
0.58	0.017	0.017	0.0199	0.017	0.018
0.62	0.0086	0.0086	0.0086	0.0086	0.009



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 10

TABLE 5-4

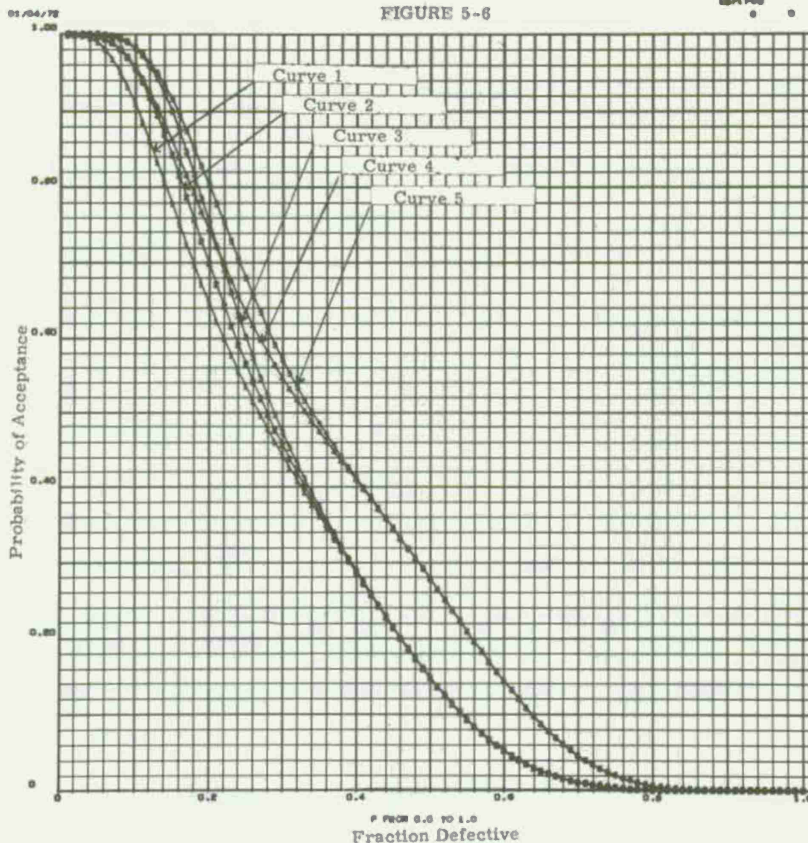
Fraction Defective $P$	Probability of Acceptance				
	Curve 1 $K_1=2, C_1=0$	Curve 2 $K_1=2, C_1=0$	Curve 3 $K_1=2, C_1=0$	Curve 4 $K_1=2, C_1=0$	Curve 5 $K_1=2, C_1=0$
	$K_2=3, C_2=1$	$K_2=3, C_2=2$	$K_2=3, C_2=3$	$K_2=3, C_2=4$	$K_2=3, C_2=5$
0.01	0.96	0.997	1.0	1.0	1.0
0.02	0.93	0.98	0.997	1.0	1.0
0.03	0.86	0.95	0.99	0.997	1.0
0.04	0.79	0.89	0.96	0.99	0.998
0.05	0.71	0.82	0.92	0.97	0.99
0.06	0.64	0.75	0.86	0.94	0.98
0.07	0.57	0.67	0.78	0.89	0.96
0.08	0.51	0.59	0.69	0.81	0.91
0.09	0.45	0.52	0.61	0.73	0.85
0.10	0.396	0.45	0.53	0.64	0.77
0.11	0.35	0.39	0.46	0.55	0.67
0.12	0.31	0.34	0.39	0.36	0.57
0.13	0.27	0.2998	0.34	0.39	0.48
0.14	0.24	0.26	0.29	0.33	0.396
0.15	0.21	0.23	0.25	0.28	0.33
0.16	0.19	0.198	0.21	0.24	0.27
0.17	0.16	0.17	0.18	0.20	0.23
0.18	0.14	0.15	0.16	0.17	0.19
0.19	0.13	0.14	0.14	0.15	0.16
0.20	0.11	0.11	0.12	0.13	0.13
0.21	0.097	0.10	0.10	0.11	0.11
0.22	0.085	0.087	0.090	0.094	0.098
0.26	0.0497	0.050	0.051	0.052	0.053
0.31	0.025	0.025	0.025	0.025	0.025
0.35	0.013	0.014	0.014	0.014	0.014
0.38	0.0084	0.0084	0.0084	0.0084	0.0085



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 10

TABLE 5-5

Fraction Defective  P	Probability of Acceptance				
	Curve 1 $K_1=2 \ C_1=1$	Curve 2 $K_1=2 \ C_1=1$	Curve 3 $K_1=2 \ C_1=1$	Curve 4 $K_1=2 \ C_1=1$	Curve 5 $K_1=2 \ C_1=1$
	$K_2=3 \ C_2=2$	$K_2=3 \ C_2=3$	$K_2=3 \ C_2=4$	$K_2=3 \ C_2=5$	$K_2=3 \ C_2=6$
0.01	0.998	1.0	1.0	1.0	1.0
0.02	0.99	0.998	1.0	1.0	1.0
0.03	0.97	0.99	0.998	1.0	1.0
0.04	0.94	0.98	0.99	0.999	1.0
0.05	0.899	0.96	0.99	0.997	0.999
0.06	0.86	0.93	0.97	0.99	0.998
0.07	0.81	0.89	0.95	0.98	0.99
0.08	0.77	0.85	0.92	0.97	0.99
0.09	0.72	0.81	0.88	0.94	0.98
0.10	0.68	0.76	0.84	0.91	0.96
0.11	0.64	0.71	0.79	0.87	0.93
0.12	0.59	0.66	0.74	0.82	0.898
0.13	0.55	0.61	0.68	0.76	0.85
0.14	0.52	0.57	0.63	0.71	0.796
0.17	0.41	0.45	0.48	0.54	0.61
0.19	0.35	0.38	0.40	0.44	0.49
0.21	0.30	0.32	0.34	0.36	0.39
0.23	0.26	0.27	0.28	0.29	0.31
0.25	0.22	0.22	0.23	0.24	0.25
0.27	0.18	0.19	0.19	0.195	0.20
0.29	0.15	0.15	0.16	0.16	0.16
0.31	0.12	0.13	0.13	0.13	0.13
0.33	0.10	0.10	0.10	0.10	0.10
0.35	0.081	0.082	0.082	0.083	0.083
0.37	0.065	0.065	0.065	0.065	0.066
0.39	0.051	0.051	0.051	0.051	0.051
0.41	0.039	0.039	0.0395	0.0397	0.0398
0.46	0.0198	0.0198	0.0198	0.0198	0.0198
0.51	0.0090	0.0090	0.0090	0.0090	0.0090



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 10

TABLE 5-6

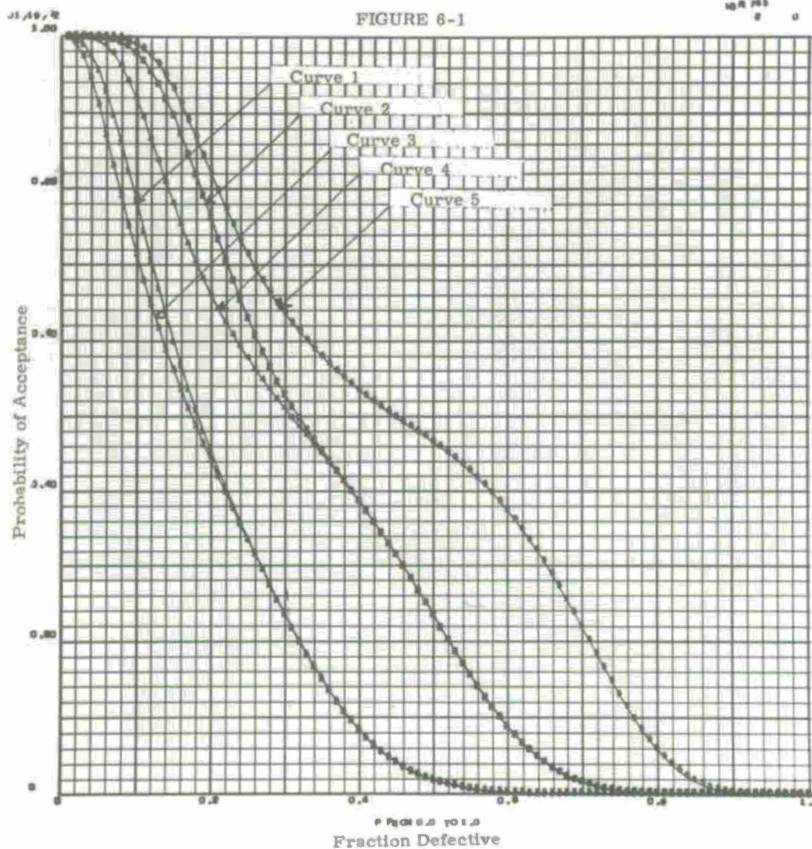
Fraction Defective  P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=4$	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=5$	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=6$	$K_1=2 \ C_1=4$ $K_2=3 \ C_2=5$	$K_1=2 \ C_1=4$ $K_2=3 \ C_2=6$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	1.0	1.0	1.0	1.0	1.0
0.03	0.990	1.0	1.0	1.0	1.0
0.04	0.996	0.999	1.0	0.999	1.0
0.05	0.99	0.998	1.0	0.998	1.0
0.06	0.98	0.99	0.999	0.99	0.999
0.07	0.97	0.99	0.997	0.99	0.997
0.08	0.95	0.98	0.99	0.98	0.99
0.09	0.93	0.97	0.99	0.97	0.99
0.10	0.91	0.96	0.98	0.96	0.98
0.11	0.89	0.94	0.97	0.94	0.97
0.12	0.86	0.92	0.96	0.92	0.96
0.13	0.83	0.89	0.94	0.91	0.95
0.14	0.81	0.87	0.92	0.88	0.93
0.15	0.78	0.84	0.899	0.86	0.92
0.16	0.75	0.81	0.87	0.84	0.896
0.20	0.65	0.699	0.75	0.74	0.80
0.24	0.56	0.59	0.63	0.66	0.70
0.29	0.46	0.48	0.497	0.562	0.59
0.31	0.43	0.44	0.45	0.53	0.55
0.34	0.38	0.38	0.39	0.49	0.4999
0.37	0.33	0.33	0.34	0.45	0.45
0.41	0.27	0.27	0.27	0.396	0.399
0.44	0.23	0.23	0.23	0.36	0.36
0.48	0.17	0.17	0.17	0.30	0.30
0.55	0.093	0.093	0.093	0.21	0.21
0.59	0.059	0.059	0.059	0.16	0.16
0.63	0.034	0.034	0.034	0.11	0.11

SECTION 6.0

CHAIN SAMPLING PLANS -  $n=15$







CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-1

Fraction Defective $P$	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=0 \ C_1=0$	$K_1=0 \ C_1=0$	$K_1=0 \ C_1=0$	$K_1=0 \ C_1=0$	$K_1=0 \ C_1=0$
	$K_2=2 \ C_2=3$	$K_2=2 \ C_2=6$	$K_2=3 \ C_2=3$	$K_2=3 \ C_2=6$	$K_2=3 \ C_2=9$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.998	1.0	0.992	1.0	1.0
0.03	0.99	1.0	0.97	1.0	1.0
0.04	0.98	1.0	0.95	0.998	1.0
0.05	0.96	0.999	0.91	0.995	1.0
0.06	0.93	0.998	0.87	0.99	1.0
0.07	0.897	0.996	0.83	0.98	0.999
0.08	0.86	0.99	0.79	0.96	0.998
0.09	0.82	0.99	0.75	0.94	0.996
0.10	0.78	0.98	0.71	0.92	0.99
0.11	0.74	0.97	0.68	0.89	0.99
0.12	0.71	0.95	0.65	0.87	0.98
0.13	0.67	0.94	0.62	0.84	0.96
0.14	0.63	0.92	0.59	0.81	0.95
0.21	0.43	0.73	0.42	0.64	0.797
0.23	0.38	0.68	0.38	0.61	0.75
0.25	0.34	0.63	0.33	0.58	0.71
0.29	0.25	0.55	0.25	0.52	0.65
0.33	0.18	0.48	0.18	0.47	0.60
0.38	0.11	0.41	0.11	0.41	0.55
0.45	0.041	0.31	0.041	0.31	0.498
0.52	0.012	0.20	0.012	0.20	0.45
0.58	0.0032	0.11	0.0032	0.11	0.397
0.62		0.066	0.0011	0.066	0.35
0.65		0.040	0.00048	0.040	0.30
0.71		0.012	0.000063	0.012	0.20
0.77		0.0023		0.0023	0.097
0.80					0.058
0.83					0.029
0.87					0.0084

FIGURE 6-2

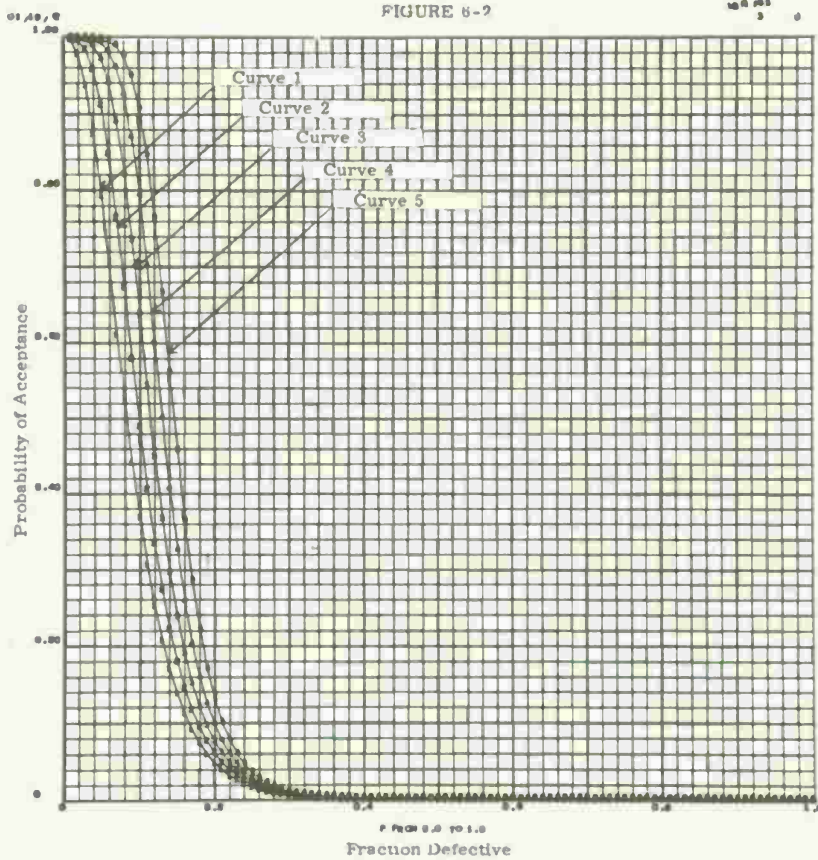
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-2

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=2$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=3$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=4$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=5$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=6$
0.01	0.997	1.0	1.0	1.0	1.0
0.02	0.98	0.997	1.0	1.0	1.0
0.03	0.94	0.99	0.997	1.0	1.0
0.04	0.87	0.96	0.99	0.998	1.0
0.05	0.79	0.91	0.97	0.99	0.999
0.06	0.70	0.85	0.94	0.98	0.996
0.07	0.61	0.77	0.89	0.96	0.99
0.08	0.52	0.67	0.82	0.93	0.98
0.09	0.44	0.58	0.73	0.87	0.95
0.10	0.37	0.49	0.64	0.79	0.91
0.11	0.31	0.41	0.54	0.70	0.85
0.12	0.25	0.33	0.45	0.60	0.76
0.13	0.21	0.27	0.37	0.4997	0.67
0.14	0.17	0.22	0.29	0.41	0.56
0.15	0.14	0.18	0.24	0.32	0.46
0.16	0.11	0.14	0.19	0.26	0.36
0.17	0.092	0.12	0.15	0.20	0.28
0.18	0.074	0.094	0.12	0.16	0.22
0.19	0.060	0.075	0.095	0.12	0.17
0.20	0.049	0.061	0.075	0.096	0.13
0.21	0.039	0.049	0.0598	0.075	0.099
0.22	0.032	0.039	0.048	0.059	0.076
0.23	0.025	0.031	0.038	0.046	0.058
0.24	0.020	0.025	0.030	0.036	0.045
0.25	0.016	0.0197	0.024	0.029	0.035
0.26	0.013	0.016	0.019	0.022	0.027
0.27	0.011	0.012	0.015	0.018	0.021
0.28	0.0084	0.0099	0.012	0.014	0.016
0.29	0.0067	0.0078	0.0093	0.011	0.013
0.30	0.0053	0.0062	0.0073	0.0086	0.0097



FIGURE 6-3

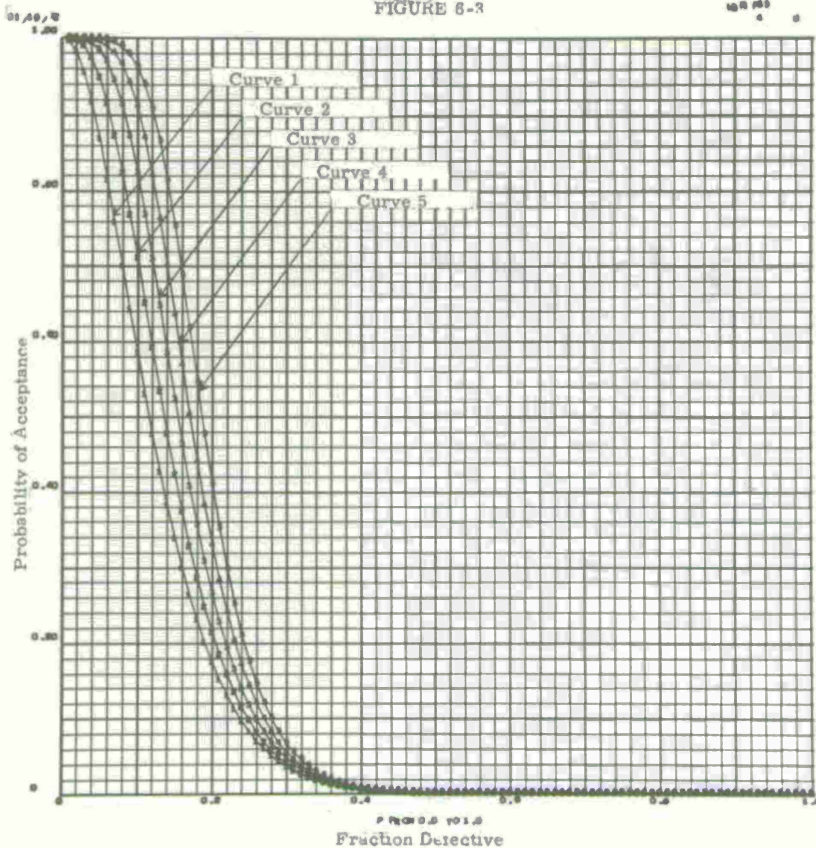
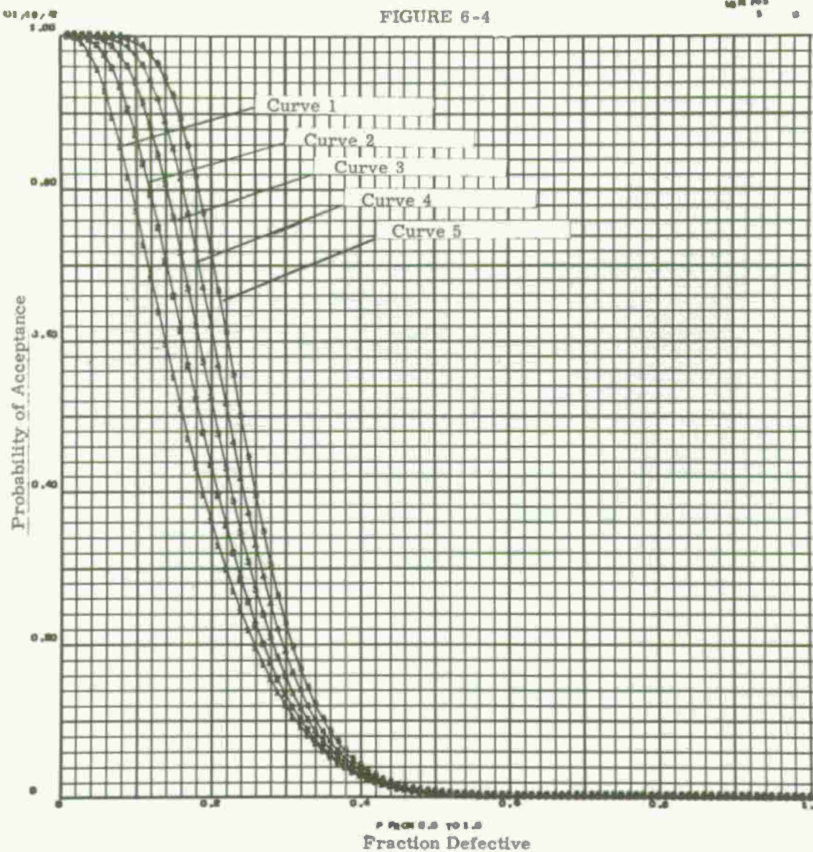
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-3

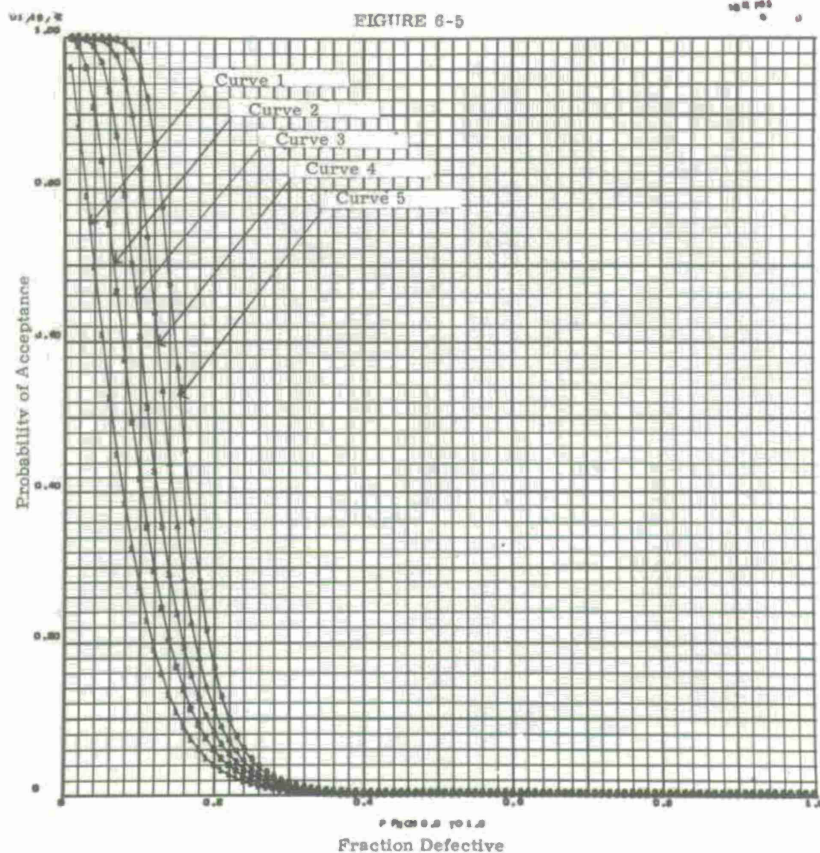
Fraction Defective p	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=2$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=3$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=4$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=5$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=6$
0.01	0.997	1.0	1.0	1.0	1.0
0.02	0.98	0.997	1.0	1.0	1.0
0.03	0.96	0.99	0.998	1.0	1.0
0.04	0.92	0.97	0.99	0.999	1.0
0.05	0.87	0.95	0.98	0.997	1.0
0.06	0.82	0.91	0.97	0.99	0.998
0.07	0.76	0.87	0.95	0.98	0.995
0.08	0.70	0.82	0.91	0.97	0.99
0.09	0.64	0.77	0.87	0.94	0.98
0.10	0.59	0.71	0.82	0.91	0.96
0.11	0.53	0.65	0.77	0.87	0.94
0.12	0.48	0.59	0.71	0.82	0.91
0.13	0.43	0.53	0.65	0.76	0.87
0.14	0.38	0.48	0.58	0.70	0.81
0.15	0.34	0.42	0.52	0.63	0.75
0.17	0.26	0.33	0.41	0.50	0.62
0.19	0.20	0.25	0.31	0.38	0.48
0.20	0.17	0.21	0.26	0.32	0.41
0.22	0.13	0.16	0.19	0.24	0.296
0.24	0.096	0.11	0.14	0.17	0.21
0.26	0.06997	0.080	0.097	0.12	0.14
0.28	0.050	0.057	0.067	0.081	0.098
0.30	0.036	0.039	0.046	0.055	0.066
0.32	0.025	0.027	0.031	0.037	0.044
0.34	0.017	0.018	0.021	0.024	0.029
0.36	0.012	0.012	0.014	0.016	0.018
0.38	0.0079	0.0081	0.0088	0.010	0.012
0.40	0.0052			0.0064	0.0074



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-4

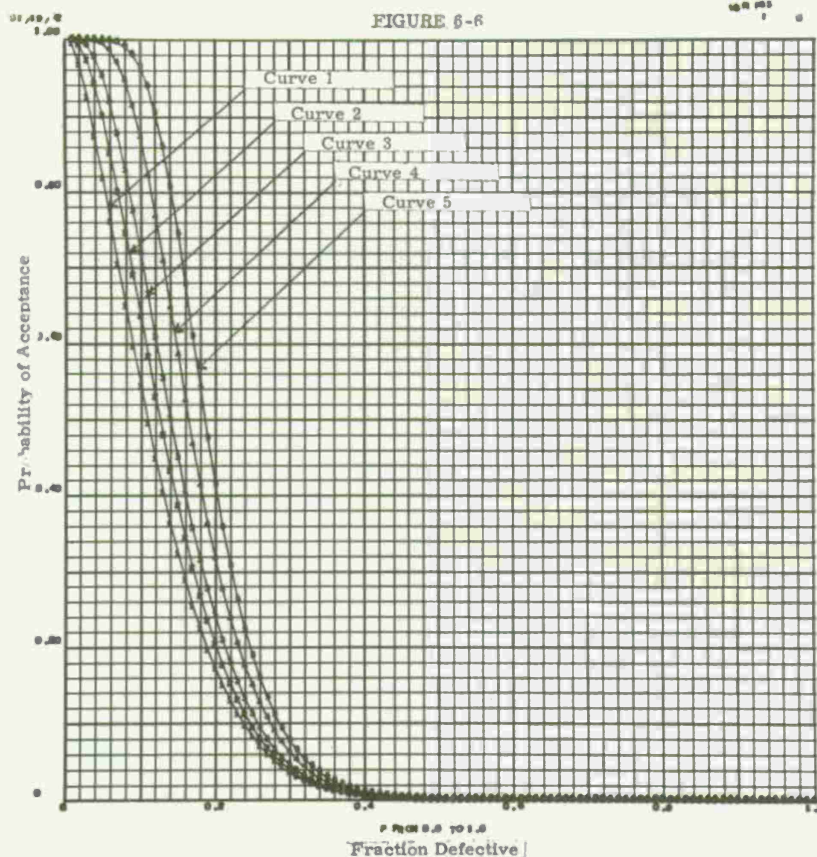
Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=3$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=4$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=5$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=6$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=7$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.998	1.0	1.0	1.0	1.0
0.03	0.99	0.998	1.0	1.0	1.0
0.04	0.98	0.99	1.0	1.0	1.0
0.05	0.96	0.99	0.997	0.999	1.0
0.06	0.93	0.97	0.99	0.998	1.0
0.07	0.89	0.96	0.99	0.996	0.999
0.08	0.86	0.93	0.97	0.99	0.998
0.09	0.81	0.90	0.96	0.99	0.996
0.10	0.77	0.87	0.94	0.98	0.99
0.11	0.73	0.83	0.91	0.96	0.99
0.12	0.68	0.79	0.88	0.94	0.98
0.13	0.64	0.75	0.84	0.92	0.96
0.14	0.59	0.70	0.80	0.89	0.94
0.15	0.55	0.66	0.76	0.85	0.92
0.16	0.51	0.61	0.72	0.81	0.89
0.17	0.47	0.57	0.67	0.77	0.86
0.18	0.43	0.52	0.62	0.72	0.81
0.20	0.36	0.43	0.52	0.62	0.72
0.22	0.298	0.36	0.43	0.51	0.61
0.24	0.24	0.28	0.34	0.41	0.497
0.26	0.19	0.22	0.27	0.33	0.39
0.27	0.17	0.198	0.24	0.29	0.35
0.28	0.15	0.17	0.21	0.25	0.30
0.30	0.12	0.13	0.16	0.19	0.23
0.32	0.092	0.10	0.12	0.14	0.17
0.34	0.069	0.074	0.084	0.099	0.12
0.36	0.051	0.054	0.060	0.070	0.083



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15  
TABLE 6-5

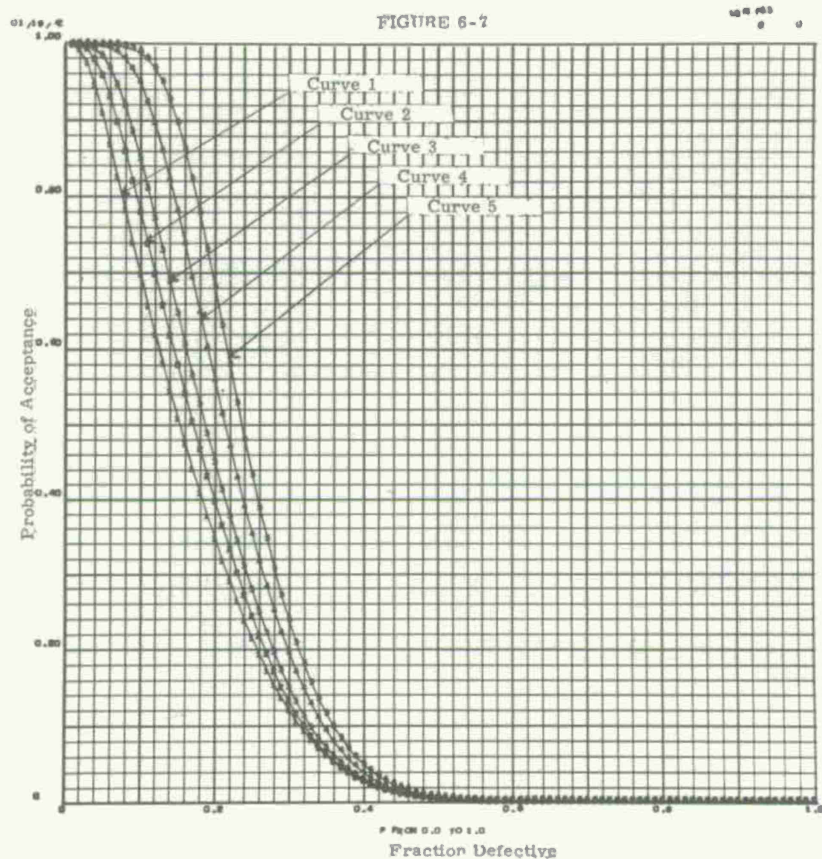
Fraction Defective  P	Probability of Acceptance				
	Curve 1 K <sub>1</sub> =1 C <sub>1</sub> =0	Curve 2 K <sub>1</sub> =1 C <sub>1</sub> =0	Curve 3 K <sub>1</sub> =1 C <sub>1</sub> =0	Curve 4 K <sub>1</sub> =1 C <sub>1</sub> =0	Curve 5 K <sub>1</sub> =1 C <sub>1</sub> =0
	K <sub>2</sub> =3 C <sub>2</sub> =1	K <sub>2</sub> =3 C <sub>2</sub> =3	K <sub>2</sub> =3 C <sub>2</sub> =5	K <sub>2</sub> =3 C <sub>2</sub> =7	K <sub>2</sub> =3 C <sub>2</sub> =9
0.01	0.96	1.0	1.0	1.0	1.0
0.02	0.88	0.99	1.0	1.0	1.0
0.03	0.79	0.96	0.997	1.0	1.0
0.04	0.697	0.91	0.99	1.0	1.0
0.05	0.61	0.84	0.97	0.997	1.0
0.06	0.53	0.75	0.93	0.99	0.999
0.07	0.45	0.66	0.87	0.98	0.998
0.08	0.38	0.58	0.79	0.95	0.99
0.09	0.32	0.49	0.70	0.897	0.98
0.10	0.27	0.42	0.60	0.83	0.96
0.11	0.23	0.35	0.51	0.74	0.92
0.12	0.19	0.29	0.43	0.64	0.86
0.13	0.16	0.24	0.35	0.53	0.78
0.14	0.13	0.20	0.29	0.44	0.67
0.15	0.11	0.17	0.23	0.35	0.56
0.16	0.088	0.14	0.19	0.28	0.45
0.17	0.072	0.11	0.15	0.22	0.36
0.18	0.059	0.090	0.13	0.18	0.28
0.19	0.049	0.073	0.10	0.14	0.21
0.20	0.0398	0.059	0.081	0.11	0.16
0.21	0.033	0.048	0.066	0.087	0.13
0.22	0.027	0.038	0.053	0.069	0.096
0.23	0.022	0.031	0.042	0.055	0.074
0.24	0.018	0.025	0.034	0.043	0.057
0.25	0.014	0.0197	0.027	0.034	0.044
0.26	0.012	0.016	0.022	0.027	0.035
0.27	0.0093	0.012	0.017	0.022	0.027





CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15  
TABLE 6-6

Fraction Defective  P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=1$ $K_2=3 \ C_2=2$	$K_1=1 \ C_1=1$ $K_2=3 \ C_2=3$	$K_1=1 \ C_1=1$ $K_2=3 \ C_2=4$	$K_1=1 \ C_1=1$ $K_2=3 \ C_2=6$	$K_1=1 \ C_1=1$ $K_2=3 \ C_2=8$
0.01	0.99	1.0	1.0	1.0	1.0
0.02	0.97	0.99	0.998	1.0	1.0
0.03	0.92	0.97	0.99	1.0	1.0
0.04	0.87	0.94	0.98	0.998	1.0
0.05	0.82	0.899	0.95	0.99	1.0
0.06	0.76	0.85	0.92	0.99	0.999
0.07	0.70	0.799	0.88	0.97	0.996
0.08	0.65	0.74	0.83	0.95	0.99
0.09	0.595	0.69	0.78	0.91	0.98
0.10	0.54	0.63	0.72	0.87	0.96
0.11	0.49	0.58	0.66	0.82	0.94
0.12	0.45	0.53	0.61	0.77	0.90
0.13	0.40	0.48	0.55	0.71	0.86
0.14	0.36	0.43	0.50	0.64	0.81
0.15	0.32	0.39	0.45	0.58	0.74
0.16	0.29	0.34	0.40	0.52	0.68
0.17	0.25	0.30	0.36	0.47	0.61
0.18	0.22	0.27	0.31	0.41	0.54
0.20	0.17	0.20	0.24	0.32	0.41
0.22	0.13	0.15	0.18	0.24	0.31
0.24	0.095	0.11	0.13	0.17	0.22
0.26	0.0697	0.079	0.093	0.12	0.16
0.28	0.050	0.056	0.065	0.088	0.11
0.30	0.036	0.039	0.045	0.061	0.076
0.32	0.025	0.027	0.031	0.041	0.052
0.34	0.017	0.018	0.020	0.028	0.035
0.36	0.012	0.012	0.014	0.018	0.023
0.38	0.0079	0.0081	0.0088	0.012	0.015
0.40	0.0052		0.0057	0.0073	0.0095

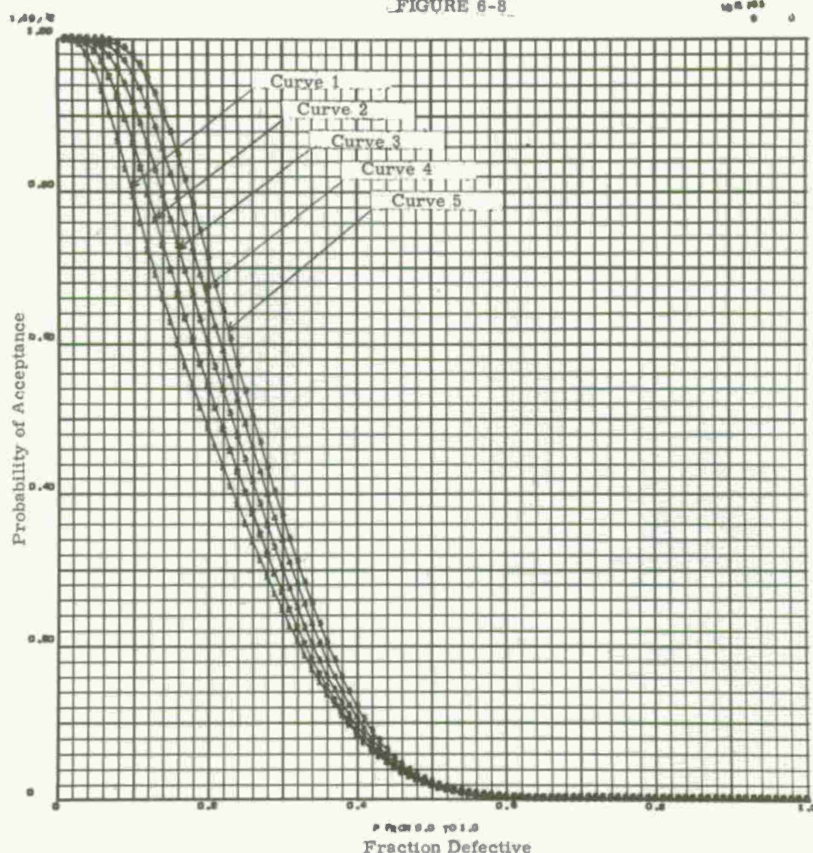


CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-7

Fraction Defective  P	Probability of Acceptance				
	Curve 1 $K_1=1 \ C_1=2$	Curve 2 $K_1=1 \ C_1=2$	Curve 3 $K_1=1 \ C_1=2$	Curve 4 $K_1=1 \ C_1=2$	Curve 5 $K_1=1 \ C_1=2$
	$K_2=3 \ C_2=3$	$K_2=3 \ C_2=4$	$K_2=3 \ C_2=5$	$K_2=3 \ C_2=7$	$K_2=3 \ C_2=9$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.99	0.999	1.0	1.0	1.0
0.03	0.97	0.99	0.998	1.0	1.0
0.04	0.95	0.98	0.99	1.0	1.0
0.05	0.91	0.96	0.98	0.999	1.0
0.06	0.87	0.93	0.97	0.996	1.0
0.07	0.83	0.897	0.95	0.99	0.999
0.08	0.78	0.86	0.92	0.98	0.998
0.09	0.74	0.82	0.89	0.97	0.99
0.10	0.698	0.78	0.85	0.95	0.99
0.11	0.66	0.74	0.81	0.92	0.98
0.12	0.62	0.698	0.77	0.895	0.97
0.13	0.58	0.66	0.73	0.86	0.95
0.14	0.54	0.62	0.69	0.82	0.93
0.15	0.51	0.58	0.65	0.78	0.898
0.16	0.47	0.54	0.61	0.74	0.86
0.17	0.44	0.50	0.57	0.69	0.82
0.18	0.41	0.47	0.53	0.65	0.78
0.20	0.35	0.396	0.45	0.56	0.68
0.22	0.29	0.33	0.38	0.47	0.58
0.24	0.24	0.27	0.31	0.39	0.48
0.26	0.19	0.22	0.25	0.31	0.39
0.28	0.15	0.17	0.19	0.25	0.31
0.30	0.12	0.13	0.15	0.19	0.24
0.32	0.092	0.099	0.11	0.15	0.18
0.34	0.069	0.074	0.082	0.11	0.13
0.36	0.051	0.054	0.0595	0.078	0.097
0.38	0.037	0.039	0.042	0.055	0.069
0.40	0.027	0.027	0.029	0.038	0.048

FIGURE 6-8

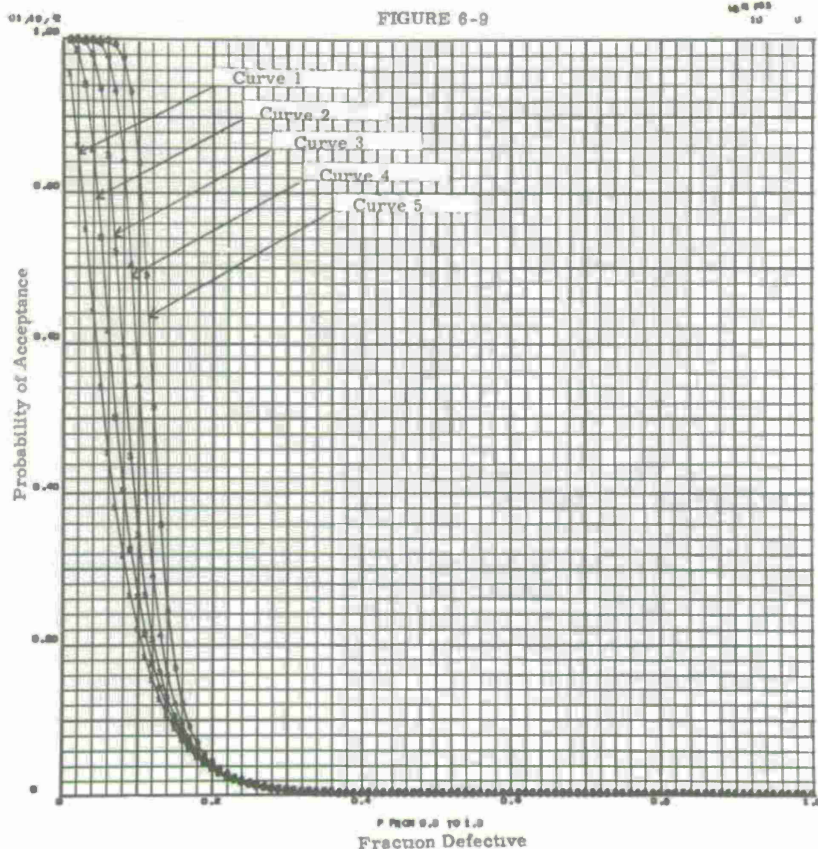


CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-8

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=4$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=5$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=6$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=7$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=8$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.999	1.0	1.0	1.0	1.0
0.03	0.99	0.998	1.0	1.0	1.0
0.04	0.98	0.99	0.998	1.0	1.0
0.05	0.96	0.98	0.995	0.999	1.0
0.06	0.93	0.97	0.99	0.996	0.999
0.07	0.90	0.95	0.98	0.99	0.997
0.08	0.87	0.92	0.96	0.98	0.99
0.09	0.83	0.89	0.94	0.97	0.99
0.10	0.79	0.86	0.92	0.96	0.98
0.11	0.76	0.83	0.89	0.93	0.97
0.12	0.72	0.796	0.86	0.91	0.95
0.13	0.69	0.76	0.83	0.88	0.93
0.14	0.66	0.73	0.79	0.85	0.90
0.18	0.54	0.60	0.66	0.72	0.78
0.21	0.46	0.51	0.57	0.62	0.68
0.24	0.38	0.43	0.48	0.52	0.57
0.30	0.25	0.27	0.30	0.34	0.37
0.36	0.14	0.14	0.16	0.18	0.20
0.38	0.11	0.11	0.12	0.14	0.16
0.40	0.084	0.087	0.094	0.10	0.12
0.42	0.064	0.066	0.0698	0.077	0.088
0.44	0.048	0.049	0.051	0.056	0.063
0.46	0.035	0.035	0.037	0.0395	0.044
0.48	0.025	0.025	0.026	0.027	0.030
0.50	0.017	0.017	0.018	0.019	0.020
0.54	0.0078	0.0078	0.0079	0.0081	0.0087
0.60	0.0019	0.0019	0.0019	0.00195	0.0020

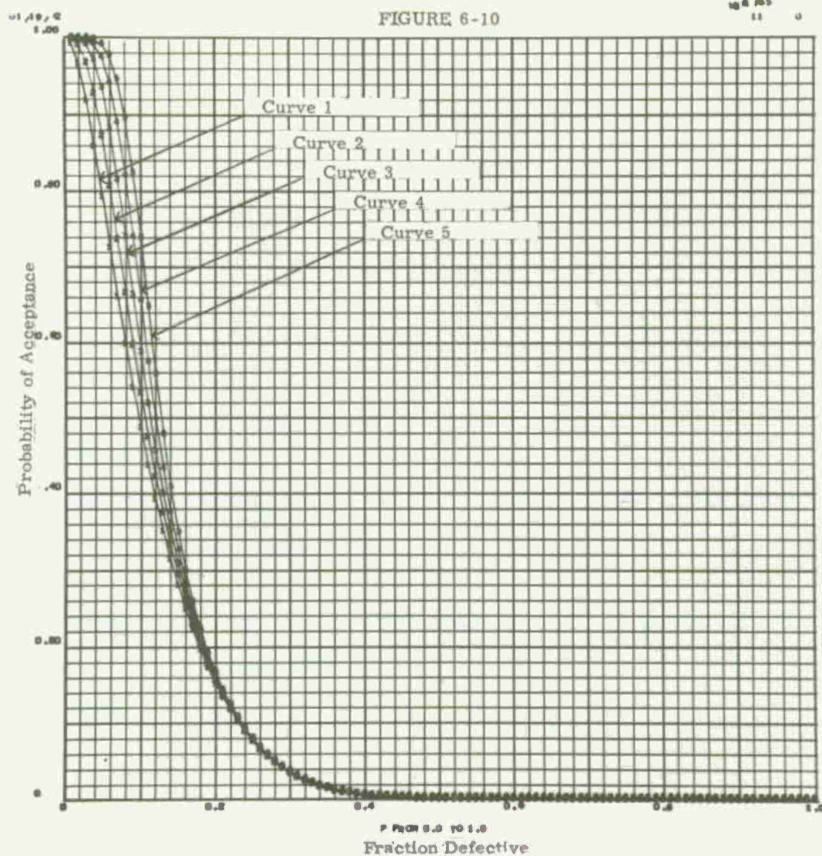




CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-9  
Probability of Acceptance

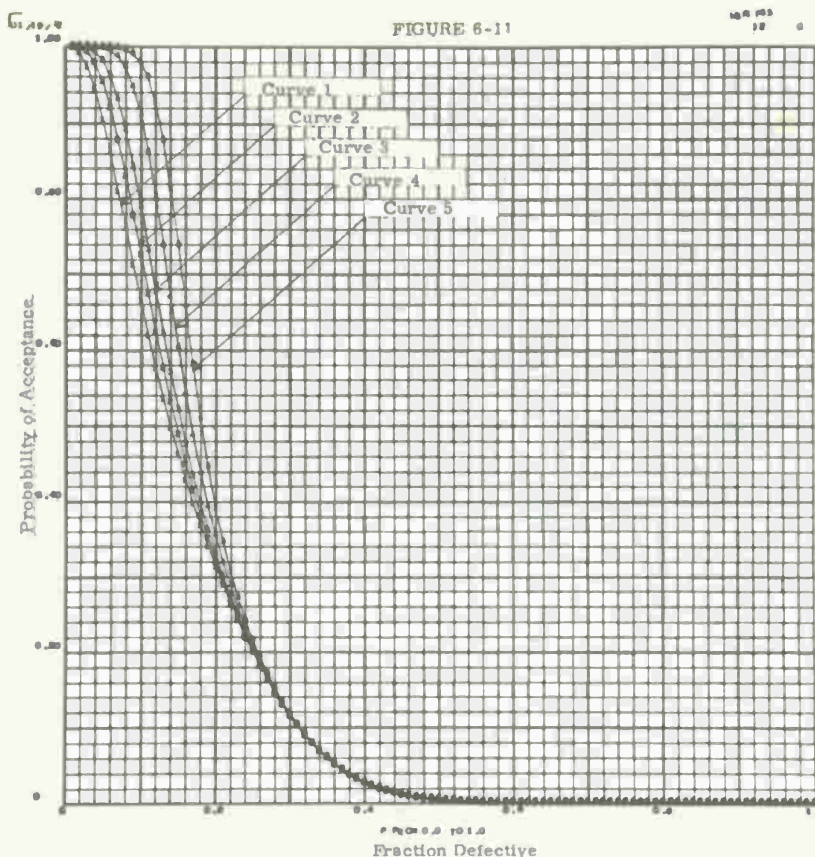
Fraction Defective P	Curve 1 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=1$	Curve 2 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=3$	Curve 3 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=5$	Curve 4 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=7$	Curve 5 $K_1=2 \ C_1=0$ $K_2=3 \ C_2=9$
0.01	0.96	0.999	1.0	1.0	1.0
0.02	0.86	0.99	1.0	1.0	1.0
0.03	0.75	0.94	0.996	1.0	1.0
0.04	0.64	0.86	0.98	0.999	1.0
0.05	0.54	0.74	0.94	0.99	1.0
0.06	0.45	0.62	0.85	0.98	0.998
0.07	0.38	0.50	0.72	0.93	0.99
0.08	0.32	0.40	0.58	0.84	0.98
0.09	0.26	0.32	0.45	0.70	0.93
0.10	0.22	0.26	0.34	0.54	0.84
0.11	0.18	0.21	0.26	0.398	0.69
0.12	0.15	0.17	0.20	0.29	0.51
0.13	0.13	0.14	0.16	0.16	0.36
0.14	0.11	0.12	0.13	0.16	0.24
0.15	0.089	0.095	0.10	0.12	0.17
0.16	0.074	0.078	0.083	0.092	0.12
0.17	0.062	0.064	0.067	0.073	0.087
0.18	0.051	0.053	0.055	0.058	0.066
0.19	0.043	0.044	0.045	0.047	0.051
0.20	0.035	0.036	0.037	0.038	0.040
0.21	0.029	0.0297	0.030	0.031	0.032
0.22	0.024	0.024	0.025	0.025	0.026
0.23	0.0199	0.020	0.020	0.021	0.021
0.24	0.016	0.016	0.017	0.017	0.017
0.25	0.013	0.013	0.014	0.014	0.014
0.26	0.011	0.011	0.011	0.011	0.011
0.27	0.0089	0.0089	0.00898	0.0090	0.0091
0.28	0.0072	0.0073	0.0073	0.0073	0.0073
0.29	0.0059	0.0059	0.0059	0.0059	0.0059
0.30	0.0047	0.0048	0.0048	0.0048	0.0048



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15  
TABLE 6-10

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=2 \ C_1=1$ $K_2=3 \ C_2=2$	$K_1=2 \ C_1=1$ $K_2=3 \ C_2=3$	$K_1=2 \ C_1=1$ $K_2=3 \ C_2=4$	$K_1=2 \ C_1=1$ $K_2=3 \ C_2=5$	$K_1=2 \ C_1=1$ $K_2=3 \ C_2=6$
0.01	0.994	0.999	1.0	1.0	1.0
0.02	0.97	0.99	0.998	1.0	1.0
0.03	0.92	0.97	0.99	0.998	1.0
0.04	0.86	0.93	0.97	0.99	0.998
0.05	0.79	0.87	0.94	0.97	0.99
0.06	0.73	0.81	0.88	0.94	0.98
0.07	0.66	0.74	0.81	0.89	0.95
0.08	0.599	0.67	0.74	0.82	0.89
0.09	0.54	0.597	0.66	0.74	0.82
0.10	0.49	0.53	0.59	0.65	0.74
0.11	0.44	0.47	0.52	0.57	0.65
0.12	0.39	0.42	0.46	0.498	0.56
0.13	0.35	0.37	0.3995	0.43	0.48
0.14	0.31	0.33	0.35	0.37	0.41
0.16	0.25	0.26	0.27	0.28	0.298
0.18	0.196	0.20	0.21	0.21	0.22
0.20	0.15	0.15	0.16	0.16	0.16
0.22	0.12	0.12	0.12	0.12	0.12
0.24	0.088	0.089	0.089	0.090	0.091
0.26	0.065	0.066	0.066	0.066	0.067
0.28	0.048	0.048	0.048	0.048	0.048
0.30	0.034	0.034	0.034	0.035	0.035
0.32	0.024	0.024	0.024	0.024	0.024
0.34	0.017	0.017	0.017	0.017	0.017





CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15  
TABLE 6-11

Fraction Defective P	Curve 1 $K_1=2 \ C_1=2$ $K_2=3 \ C_2=3$	Curve 2 $K_1=2 \ C_1=2$ $K_2=3 \ C_2=4$	Curve 3 $K_1=2 \ C_1=2$ $K_2=3 \ C_2=5$	Curve 4 $K_1=2 \ C_1=2$ $K_2=3 \ C_2=7$	Curve 5 $K_1=2 \ C_1=2$ $K_2=3 \ C_2=9$
0.01	0.999	1.0	1.0	1.0	1.0
0.02	0.99	0.998	1.0	1.0	1.0
0.03	0.97	0.99	0.998	1.0	1.0
0.04	0.94	0.98	0.99	1.0	1.0
0.05	0.90	0.95	0.98	0.998	1.0
0.06	0.86	0.92	0.96	0.995	1.0
0.07	0.81	0.88	0.93	0.99	0.999
0.08	0.76	0.83	0.89	0.97	0.997
0.09	0.71	0.78	0.84	0.95	0.99
0.10	0.66	0.73	0.79	0.91	0.98
0.11	0.62	0.67	0.73	0.86	0.96
0.12	0.58	0.62	0.68	0.80	0.93
0.13	0.54	0.58	0.62	0.74	0.88
0.14	0.497	0.53	0.57	0.67	0.81
0.16	0.43	0.45	0.47	0.54	0.66
0.18	0.37	0.38	0.396	0.44	0.51
0.20	0.31	0.32	0.33	0.35	0.39
0.22	0.26	0.27	0.27	0.29	0.30
0.24	0.22	0.22	0.22	0.23	0.24
0.26	0.18	0.18	0.18	0.19	0.19
0.28	0.14	0.14	0.15	0.15	0.15
0.30	0.11	0.11	0.11	0.12	0.12
0.32	0.089	0.089	0.089	0.089	0.089
0.34	0.068	0.068	0.068	0.068	0.068

FIGURE 6-12

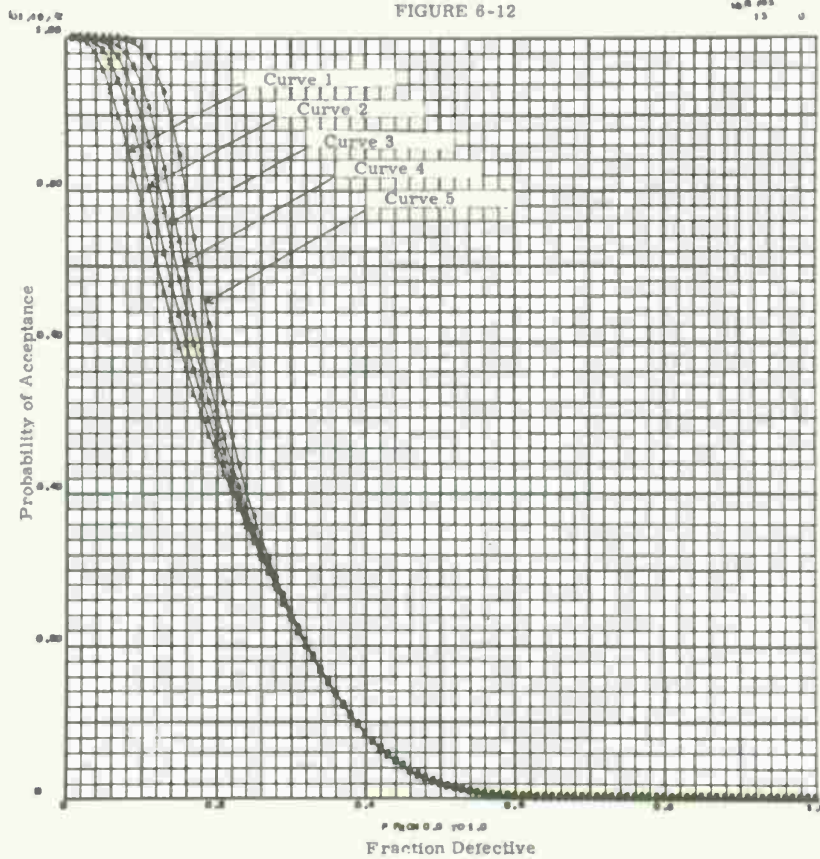
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-12

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=4$	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=5$	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=6$	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=7$	$K_1=2 \ C_1=3$ $K_2=3 \ C_2=9$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.999	1.0	1.0	1.0	1.0
0.03	0.99	0.998	1.0	1.0	1.0
0.04	0.98	0.99	0.998	1.0	1.0
0.05	0.96	0.98	0.995	0.999	1.0
0.06	0.93	0.97	0.99	0.996	1.0
0.07	0.897	0.94	0.97	0.99	0.999
0.08	0.86	0.92	0.96	0.98	0.997
0.09	0.82	0.88	0.93	0.96	0.994
0.10	0.78	0.84	0.897	0.94	0.99
0.11	0.74	0.80	0.86	0.91	0.98
0.12	0.70	0.76	0.82	0.87	0.96
0.13	0.66	0.72	0.77	0.83	0.93
0.14	0.63	0.68	0.73	0.78	0.89
0.16	0.56	0.597	0.64	0.68	0.79
0.18	0.50	0.53	0.56	0.59	0.68
0.20	0.45	0.47	0.49	0.51	0.57
0.22	0.40	0.41	0.42	0.44	0.47
0.24	0.36	0.36	0.37	0.38	0.3998
0.26	0.31	0.32	0.32	0.33	0.34
0.28	0.27	0.28	0.28	0.28	0.29
0.30	0.23	0.24	0.24	0.24	0.24
0.32	0.198	0.199	0.199	0.20	0.20
0.34	0.16	0.16	0.16	0.17	0.17
0.36	0.13	0.13	0.13	0.13	0.13
0.38	0.11	0.11	0.11	0.11	0.11
0.40	0.083	0.083	0.083	0.083	0.083
0.42	0.064	0.064	0.064	0.064	0.064
0.44	0.048	0.048	0.048	0.048	0.048

FIGURE 6-13

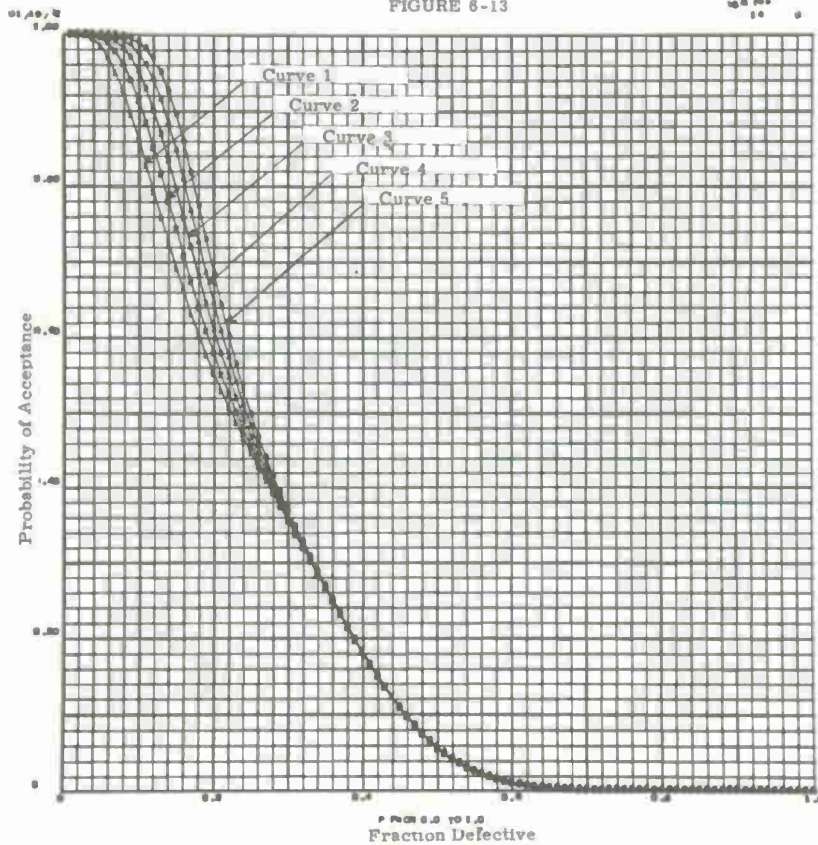
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 15

TABLE 6-13

Fraction Defective P	Probability of Acceptance				
	Curve 1 K <sub>1</sub> =2 C <sub>1</sub> =4	Curve 2 K <sub>1</sub> =2 C <sub>1</sub> =4	Curve 3 K <sub>1</sub> =2 C <sub>1</sub> =4	Curve 4 K <sub>1</sub> =2 C <sub>1</sub> =4	Curve 5 K <sub>1</sub> =2 C <sub>1</sub> =4
	K <sub>2</sub> =3 C <sub>2</sub> =5	K <sub>2</sub> =3 C <sub>2</sub> =6	K <sub>2</sub> =3 C <sub>2</sub> =7	K <sub>2</sub> =3 C <sub>2</sub> =8	K <sub>2</sub> =3 C <sub>2</sub> =9
0.01	1.0	1.0	1.0	1.0	1.0
0.02	1.0	1.0	1.0	1.0	1.0
0.03	0.998	1.0	1.0	1.0	1.0
0.04	0.994	0.998	1.0	1.0	1.0
0.05	0.98	0.995	0.999	1.0	1.0
0.06	0.97	0.99	0.996	0.999	1.0
0.07	0.95	0.98	0.99	0.997	0.999
0.08	0.92	0.96	0.98	0.99	0.998
0.09	0.89	0.94	0.97	0.99	0.995
0.10	0.86	0.91	0.95	0.98	0.99
0.11	0.83	0.88	0.93	0.96	0.98
0.12	0.79	0.85	0.8999	0.94	0.97
0.13	0.76	0.81	0.87	0.91	0.95
0.15	0.69	0.74	0.795	0.85	0.89
0.18	0.60	0.64	0.68	0.73	0.77
0.20	0.55	0.58	0.61	0.64	0.68
0.22	0.51	0.52	0.55	0.57	0.60
0.24	0.47	0.48	0.49	0.51	0.53
0.26	0.43	0.43	0.44	0.45	0.47
0.28	0.39	0.39	0.3996	0.41	0.41
0.30	0.35	0.36	0.36	0.36	0.37
0.32	0.32	0.32	0.32	0.32	0.33
0.34	0.28	0.28	0.28	0.28	0.29
0.36	0.25	0.25	0.25	0.25	0.25
0.38	0.21	0.21	0.21	0.21	0.21
0.40	0.18	0.18	0.18	0.18	0.18
0.42	0.15	0.15	0.15	0.15	0.15
0.44	0.12	0.12	0.12	0.12	0.12
0.46	0.096	0.096	0.096	0.096	0.096
0.48	0.074	0.074	0.074	0.074	0.074



SECTION 7.0

CHAIN SAMPLING PLANS -  $n=20$



FIGURE 7-1

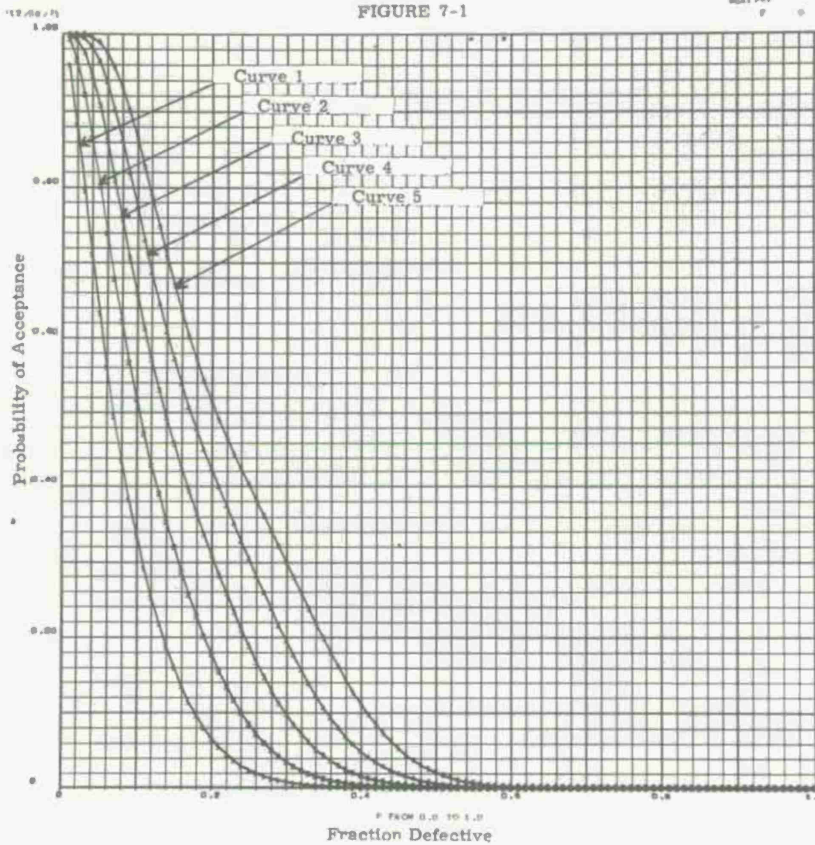
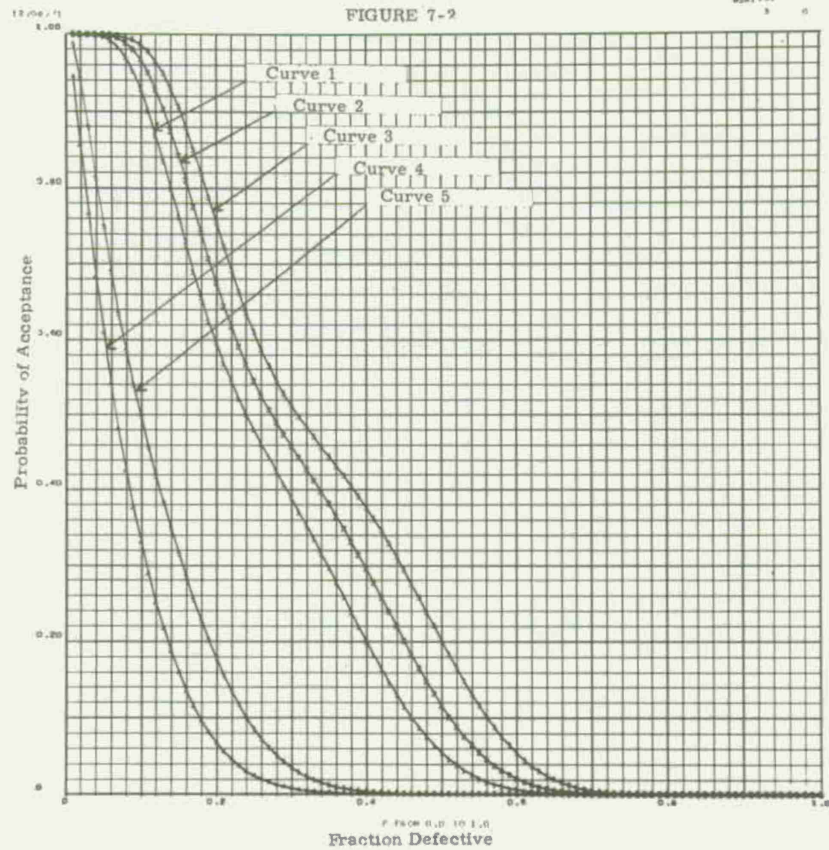
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-1

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=1$	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=2$	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=3$	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=4$	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=5$
0.01	0.96	0.994	1.0	1.0	1.0
0.02	0.88	0.967	0.99	1.0	1.0
0.03	0.80	0.92	0.98	0.994	0.999
0.04	0.71	0.86	0.95	0.98	0.996
0.05	0.63	0.80	0.91	0.96	0.99
0.06	0.56	0.74	0.86	0.94	0.98
0.07	0.49	0.68	0.81	0.90	0.96
0.08	0.44	0.62	0.76	0.86	0.93
0.09	0.38	0.57	0.71	0.82	0.90
0.1	0.33	0.52	0.66	0.77	0.87
0.11	0.29	0.47	0.61	0.73	0.83
0.12	0.25	0.43	0.57	0.69	0.79
0.14	0.19	0.35	0.49	0.61	0.71
0.16	0.13	0.29	0.42	0.54	0.63
0.18	0.10	0.23	0.36	0.48	0.57
0.20	0.07	0.18	0.31	0.42	0.51
0.24	0.03	0.10	0.21	0.33	0.42
0.28	0.012	0.05	0.13	0.24	0.34
0.32	0.005	0.02	0.07	0.16	0.26
0.34	0.003	0.015	0.051	0.12	0.22
0.36		0.01	0.035	0.092	0.18
0.38		0.006	0.024	0.068	0.14
0.40			0.016	0.049	0.11
0.45			0.005	0.019	0.05
0.50				0.006	0.02
0.55					0.006





CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-2

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=6$	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=7$	$K_1=0 \ C_1=0$ $K_2=2 \ C_2=8$	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=1$	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=2$
0.01	1.0	1.0	1.0	0.94	0.987
0.02	1.0	1.0	1.0	0.85	0.94
0.03	1.0	1.0	1.0	0.76	0.88
0.04	0.999	1.0	1.0	0.68	0.81
0.06	0.99	0.998	1.0	0.54	0.69
0.08	0.97	0.989	0.997	0.42	0.58
0.10	0.93	0.97	0.99	0.33	0.50
0.12	0.87	0.93	0.97	0.25	0.42
0.14	0.80	0.87	0.93	0.19	0.35
0.16	0.73	0.81	0.88	0.13	0.28
0.18	0.66	0.74	0.82	0.095	0.23
0.20	0.60	0.67	0.75	0.066	0.18
0.24	0.50	0.57	0.63	0.03	0.10
0.28	0.42	0.49	0.55	0.012	0.05
0.32	0.35	0.43	0.48	0.005	0.023
0.36	0.28	0.36	0.43	0.002	0.010
0.38	0.24	0.33	0.40		0.006
0.42	0.16	0.26	0.34		0.002
0.46	0.10	0.18	0.28		0.0007
0.50	0.05	0.12	0.20		
0.54	0.026	0.065	0.13		
0.58	0.011	0.031	0.075		
0.62	0.004	0.013	0.034		
0.66		0.005	0.015		
0.70		0.001	0.005		



FIGURE 7-3

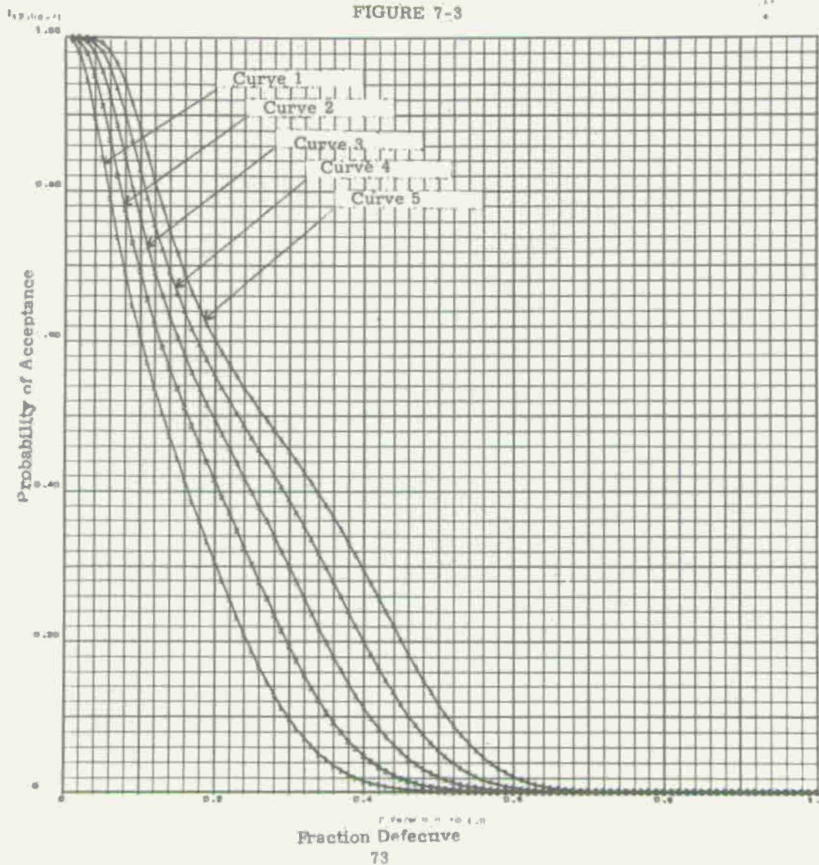
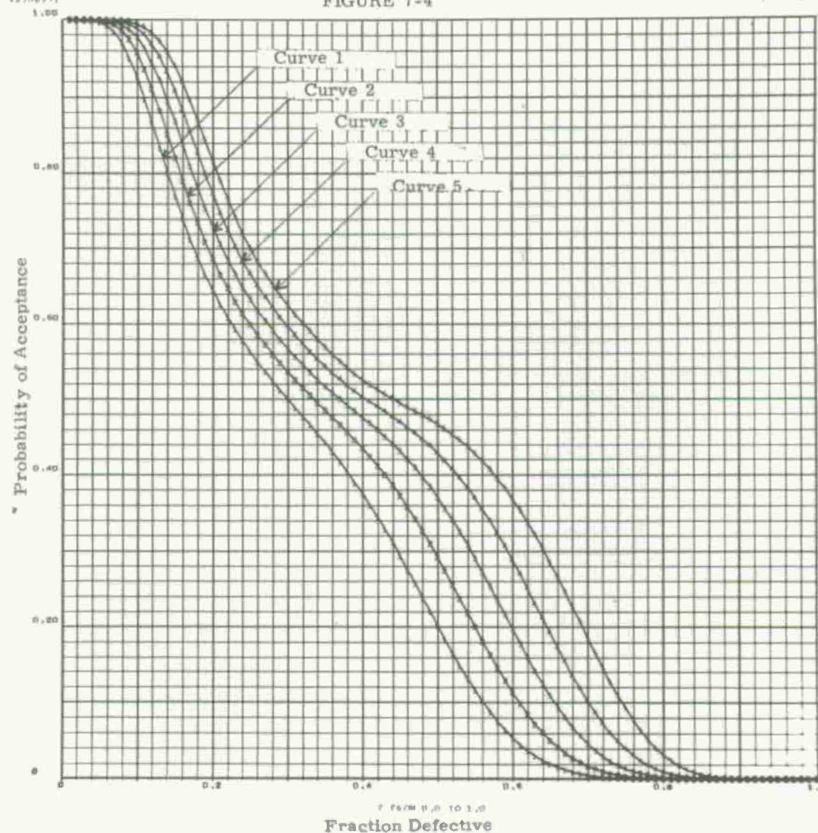
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-3

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=0$ $C_1=0$	Curve 2 $K_1=0$ $C_1=0$	Curve 3 $K_1=0$ $C_1=0$	Curve 4 $K_1=0$ $C_1=0$	Curve 5 $K_1=0$ $C_1=0$
	$K_2=3$ $C_2=3$	$K_2=3$ $C_2=4$	$K_2=3$ $C_2=5$	$K_2=3$ $C_2=6$	$K_2=3$ $C_2=7$
0.01	0.998	1.0	1.0	1.0	1.0
0.02	0.98	0.995	1.0	1.0	1.0
0.04	0.90	0.95	0.98	0.993	0.998
0.06	0.79	0.87	0.92	0.96	0.98
0.07	0.74	0.82	0.89	0.93	0.97
0.08	0.69	0.77	0.84	0.90	0.94
0.10	0.61	0.69	0.76	0.83	0.88
0.12	0.54	0.62	0.69	0.75	0.81
0.14	0.47	0.56	0.63	0.69	0.75
0.16	0.41	0.51	0.58	0.64	0.69
0.18	0.36	0.46	0.54	0.59	0.64
0.20	0.31	0.42	0.50	0.55	0.60
0.24	0.21	0.32	0.42	0.49	0.54
0.28	0.13	0.24	0.34	0.42	0.48
0.32	0.07	0.16	0.26	0.35	0.43
0.34	0.051	0.12	0.22	0.32	0.40
0.36	0.035	0.09	0.18	0.28	0.36
0.40	0.016	0.049	0.11	0.20	0.29
0.44	0.006	0.023	0.062	0.13	0.22
0.48		0.01	0.03	0.075	0.15
0.52		0.004	0.013	0.038	0.09
0.54		0.002	0.008	0.026	0.065
0.56			0.005	0.017	0.046
0.60			0.002	0.006	0.021
0.64					0.008



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-4

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=8$	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=9$	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=10$	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=11$	$K_1=0 \ C_1=0$ $K_2=3 \ C_2=12$
0.04	0.9994	1.0	1.0	1.0	1.0
0.05	0.998	0.999	1.0	1.0	1.0
0.06	0.99	0.998	0.999	1.0	1.0
0.07	0.98	0.994	0.998	0.999	1.0
0.08	0.97	0.986	0.994	0.997	0.999
0.09	0.95	0.97	0.988	0.995	0.998
0.10	0.93	0.96	0.98	0.99	0.996
0.11	0.90	0.94	0.96	0.98	0.99
0.12	0.87	0.91	0.94	0.97	0.98
0.14	0.80	0.85	0.90	0.93	0.96
0.16	0.74	0.79	0.84	0.88	0.92
0.18	0.69	0.73	0.78	0.83	0.87
0.20	0.64	0.69	0.73	0.77	0.82
0.24	0.58	0.61	0.65	0.68	0.72
0.28	0.52	0.56	0.59	0.62	0.65
0.32	0.48	0.51	0.55	0.57	0.60
0.36	0.43	0.48	0.51	0.53	0.56
0.40	0.38	0.44	0.48	0.50	0.53
0.44	0.31	0.39	0.44	0.48	0.50
0.48	0.24	0.33	0.40	0.45	0.48
0.52	0.17	0.26	0.34	0.41	0.45
0.56	0.10	0.18	0.27	0.35	0.42
0.60	0.05	0.11	0.20	0.29	0.37
0.64	0.024	0.06	0.12	0.21	0.30
0.68	0.01	0.03	0.07	0.14	0.23
0.72	0.003	0.01	0.03	0.07	0.15
0.76		0.003	0.01	0.03	0.08
0.80			0.003	0.01	0.03
0.84				0.002	0.009

FIGURE 7-5

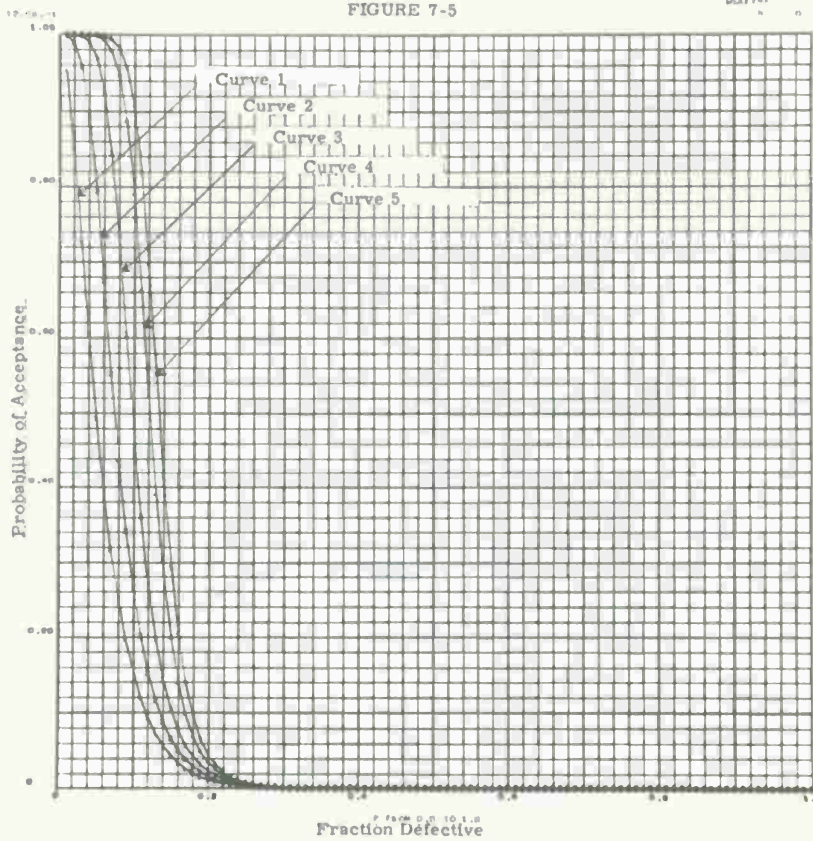
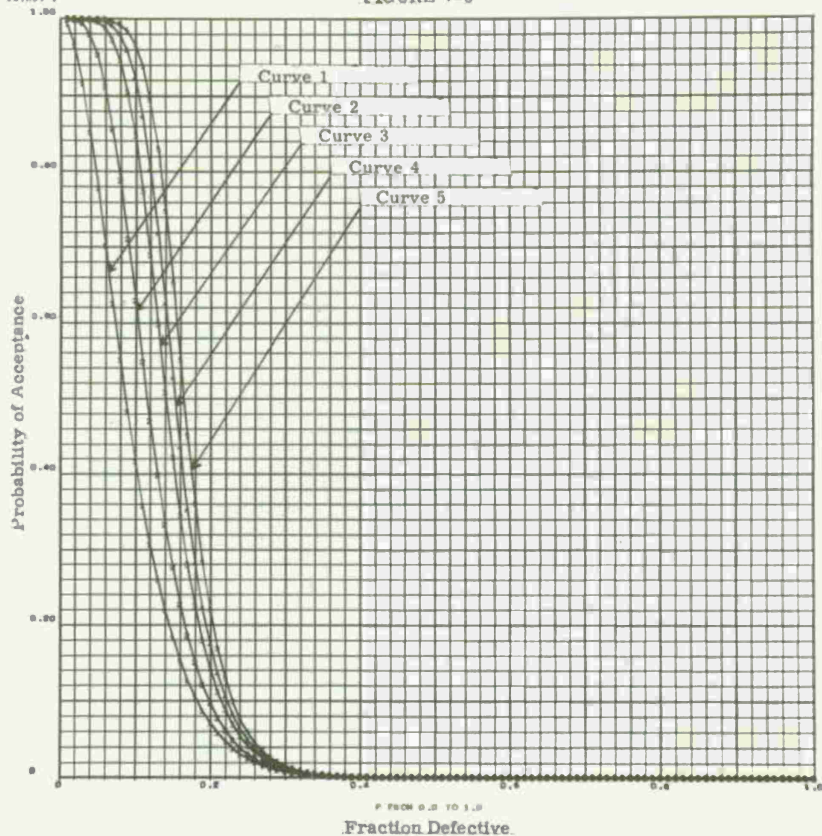
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-5

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=1$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=3$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=5$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=7$	$K_1=1 \ C_1=0$ $K_2=2 \ C_2=8$
0.01	0.95	0.999	1.0	1.0	1.0
0.02	0.85	0.99	1.0	1.0	1.0
0.03	0.73	0.96	0.998	1.0	1.0
0.04	0.60	0.89	0.99	1.0	1.0
0.05	0.49	0.79	0.97	0.998	1.0
0.06	0.40	0.68	0.92	0.99	0.998
0.07	0.32	0.55	0.84	0.98	0.994
0.08	0.25	0.44	0.73	0.95	0.98
0.09	0.20	0.34	0.60	0.89	0.96
0.10	0.15	0.26	0.47	0.79	0.90
0.11	0.12	0.20	0.36	0.66	0.82
0.12	0.09	0.15	0.27	0.52	0.69
0.13	0.07	0.12	0.20	0.39	0.55
0.14	0.06	0.09	0.14	0.28	0.41
0.15	0.04	0.07	0.10	0.20	0.29
0.16	0.034	0.05	0.08	0.14	0.20
0.17	0.026	0.04	0.06	0.1	0.14
0.18	0.02	0.03	0.04	0.07	0.09
0.19	0.016	0.02	0.03	0.05	0.06
0.20	0.012	0.016	0.02	0.03	0.04
0.21	0.009	0.012	0.017	0.02	0.03
0.22		0.009	0.013	0.018	0.02
0.23			0.01	0.013	0.016
0.24			0.007	0.009	0.01
0.25					0.008



CHAIN SAMPLING PLAN DATA

SAMPLE SIZE 20

TABLE 7-6

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=2$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=4$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=6$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=7$	$K_1=1 \ C_1=1$ $K_2=2 \ C_2=8$
0.01	0.99	1.0	1.0	1.0	1.0
0.02	0.97	0.999	1.0	1.0	1.0
0.03	0.92	0.99	1.0	1.0	1.0
0.04	0.85	0.98	0.998	1.0	1.0
0.05	0.78	0.95	0.996	0.999	1.0
0.06	0.70	0.91	0.99	0.997	0.999
0.07	0.63	0.86	0.97	0.99	0.997
0.08	0.55	0.79	0.95	0.98	0.99
0.09	0.48	0.71	0.90	0.96	0.98
0.10	0.42	0.63	0.85	0.92	0.97
0.11	0.36	0.55	0.77	0.87	0.94
0.12	0.31	0.47	0.69	0.80	0.89
0.13	0.26	0.40	0.60	0.72	0.83
0.14	0.22	0.33	0.51	0.62	0.75
0.15	0.18	0.28	0.42	0.53	0.65
0.16	0.15	0.23	0.35	0.43	0.55
0.17	0.13	0.18	0.28	0.35	0.45
0.18	0.10	0.15	0.22	0.28	0.35
0.19	0.08	0.12	0.18	0.22	0.28
0.20	0.07	0.10	0.14	0.17	0.22
0.21	0.06	0.077	0.11	0.14	0.17
0.22	0.05	0.061	0.09	0.10	0.13
0.24	0.03	0.038	0.05	0.06	0.08
0.26	0.02	0.023	0.03	0.04	0.04
0.28	0.01	0.014	0.019	0.02	0.025
0.30	0.008	0.0085	0.01	0.013	0.015
0.32			0.006	0.007	0.008



FIGURE 7-7

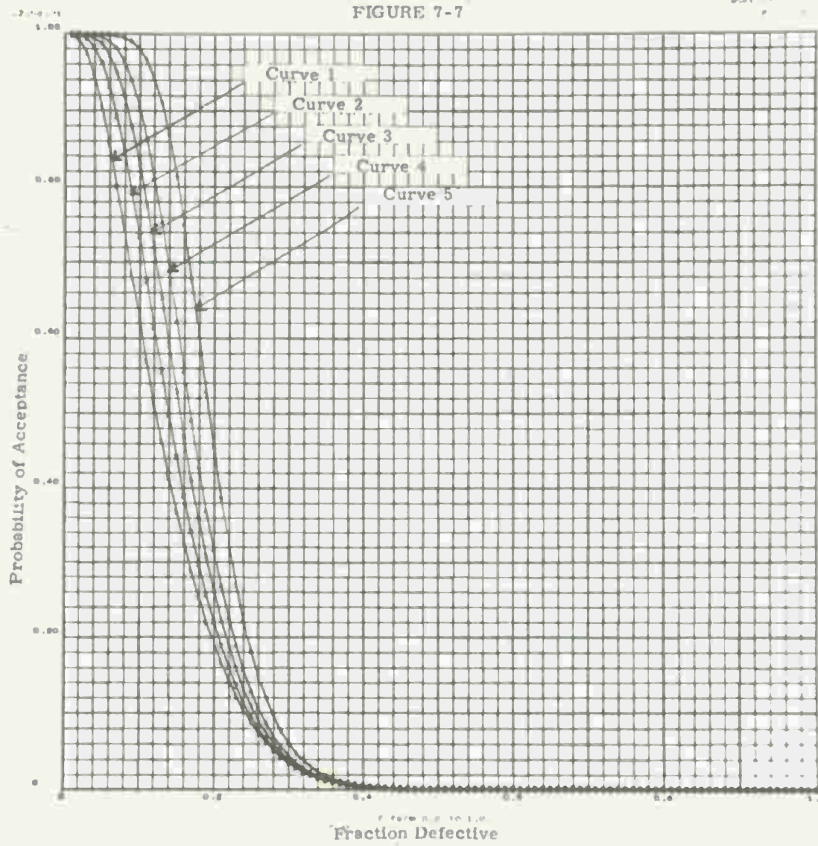
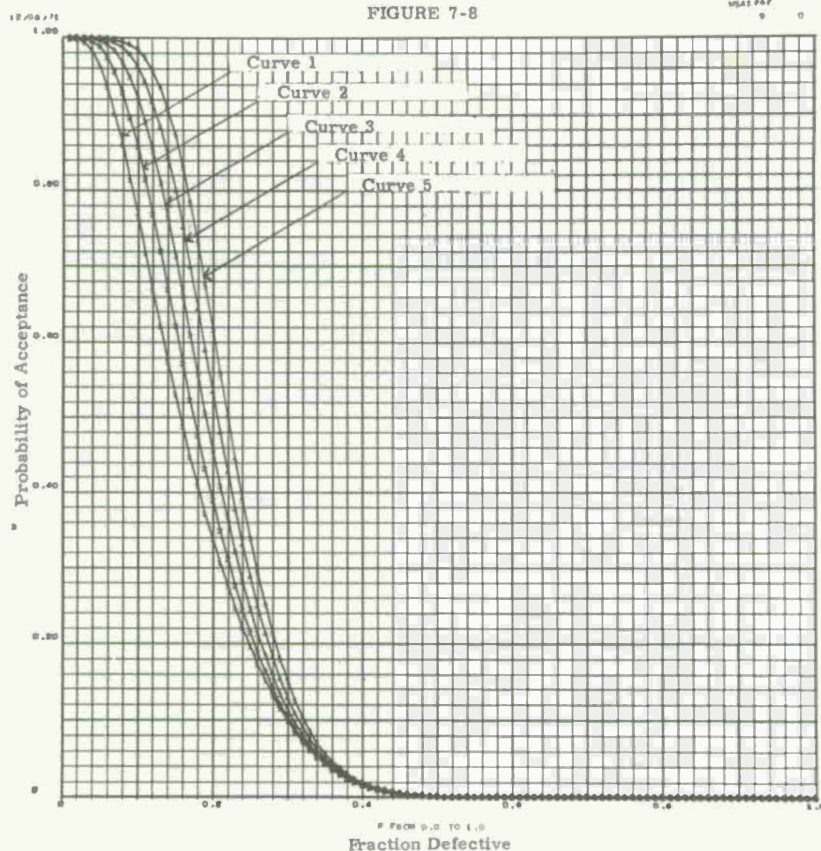
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-7

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=3$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=4$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=5$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=6$	$K_1=1 \ C_1=2$ $K_2=2 \ C_2=8$
0.01	0.999	1.0	1.0	1.0	1.0
0.03	0.98	0.99	0.999	1.0	1.0
0.04	0.95	0.98	0.996	0.999	1.0
0.05	0.90	0.96	0.99	0.997	1.0
0.06	0.85	0.93	0.97	0.99	0.999
0.07	0.80	0.89	0.95	0.98	0.998
0.08	0.74	0.84	0.92	0.96	0.996
0.09	0.68	0.79	0.88	0.94	0.99
0.10	0.63	0.73	0.83	0.90	0.98
0.11	0.57	0.68	0.78	0.86	0.97
0.12	0.51	0.62	0.72	0.81	0.94
0.13	0.46	0.56	0.65	0.75	0.91
0.14	0.41	0.50	0.59	0.69	0.87
0.15	0.37	0.44	0.53	0.62	0.81
0.16	0.33	0.39	0.47	0.55	0.75
0.17	0.29	0.34	0.41	0.49	0.68
0.19	0.22	0.26	0.31	0.37	0.53
0.21	0.17	0.19	0.22	0.27	0.38
0.23	0.12	0.13	0.16	0.19	0.27
0.25	0.09	0.10	0.11	0.13	0.18
0.27	0.06	0.07	0.07	0.08	0.12
0.29	0.04	0.04	0.05	0.06	0.08
0.31	0.028	0.03	0.03	0.04	0.048
0.33	0.019	0.019	0.02	0.02	0.029
0.35	0.012	0.012	0.013	0.014	0.018
0.36	0.008	0.0097	0.01	0.011	0.014
0.38	0.006	0.006	0.006	0.006	0.008



CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-8

F Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=1$ $C_1=3$	Curve 2 $K_1=1$ $C_1=3$	Curve 3 $K_1=1$ $C_1=3$	Curve 4 $K_1=1$ $C_1=3$	Curve 5 $K_1=1$ $C_1=3$
	$K_2=2$ $C_2=4$	$K_2=2$ $C_2=5$	$K_2=2$ $C_2=6$	$K_2=2$ $C_2=7$	$K_2=2$ $C_2=8$
0.01	1.0	1.0	1.0	1.0	1.0
0.03	0.99	0.999	1.0	1.0	1.0
0.05	0.96	0.99	0.997	0.999	1.0
0.07	0.90	0.96	0.98	0.994	0.999
0.08	0.86	0.93	0.97	0.988	0.996
0.09	0.81	0.90	0.95	0.98	0.992
0.10	0.77	0.86	0.92	0.96	0.985
0.11	0.72	0.81	0.89	0.94	0.97
0.12	0.67	0.77	0.85	0.92	0.96
0.13	0.62	0.72	0.81	0.88	0.94
0.14	0.57	0.67	0.76	0.84	0.91
0.15	0.53	0.62	0.71	0.80	0.87
0.16	0.49	0.57	0.66	0.75	0.83
0.17	0.45	0.52	0.61	0.70	0.78
0.18	0.41	0.48	0.56	0.64	0.73
0.19	0.37	0.43	0.51	0.59	0.67
0.20	0.34	0.39	0.46	0.53	0.62
0.22	0.28	0.31	0.36	0.43	0.50
0.24	0.22	0.24	0.28	0.33	0.39
0.26	0.17	0.19	0.21	0.25	0.29
0.28	0.13	0.14	0.16	0.18	0.21
0.30	0.10	0.10	0.11	0.13	0.15
0.34	0.05	0.05	0.06	0.06	0.069
0.38	0.024	0.024	0.02	0.026	0.029
0.40	0.016	0.016	0.016	0.017	0.018
0.42	0.01	0.01	0.01	0.010	0.011
0.43	0.008	0.008	0.008	0.008	0.009

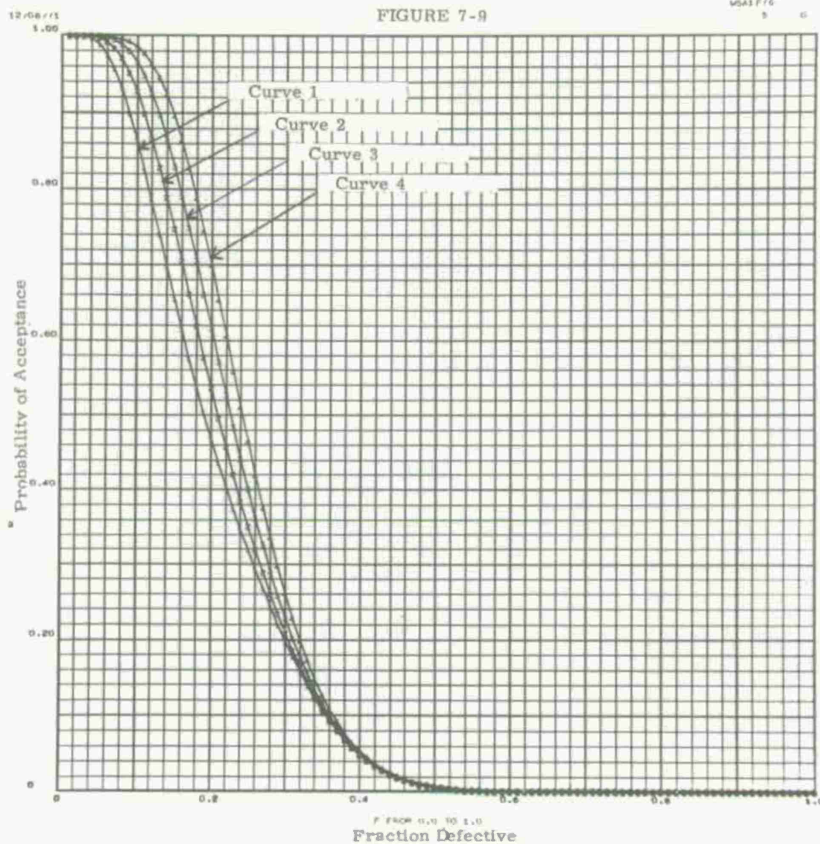
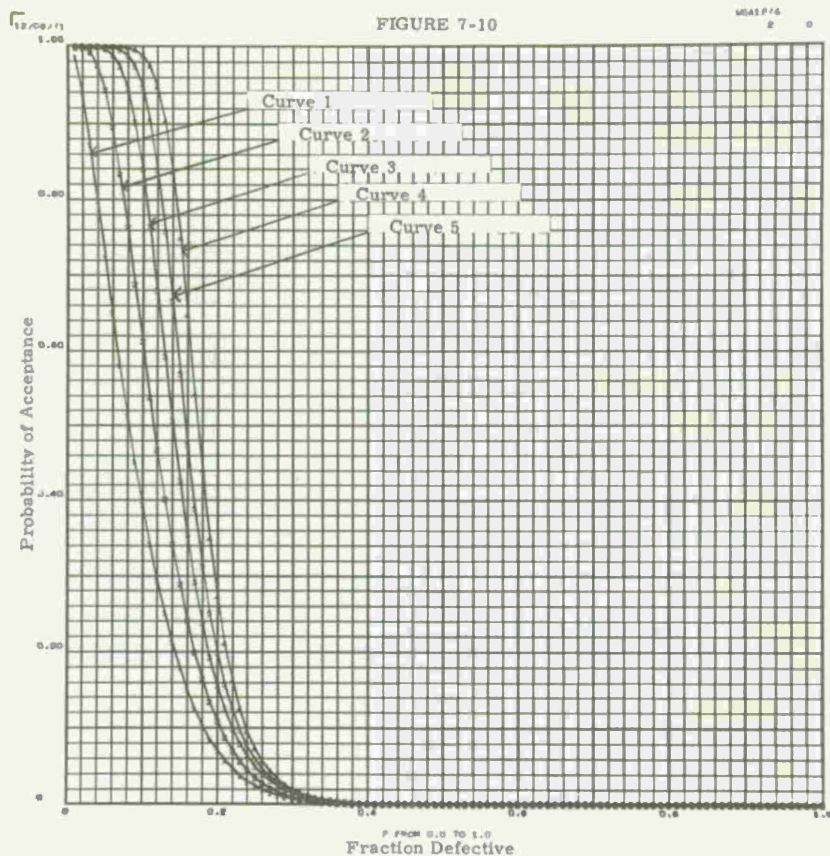


TABLE 7-9

Fraction Defective P	Probability of Acceptance			
	Curve 1 $K_1=1 \ C_1=4$	Curve 2 $K_1=1 \ C_1=4$	Curve 3 $K_1=1 \ C_1=4$	Curve 4 $K_1=1 \ C_1=4$
	$K_2=2 \ C_2=5$	$K_2=2 \ C_2=6$	$K_2=2 \ C_2=7$	$K_2=2 \ C_2=8$
0.03	0.999	1.0	1.0	1.0
0.05	0.989	0.997	0.999	1.0
0.07	0.96	0.98	0.994	0.999
0.08	0.93	0.97	0.989	0.997
0.09	0.90	0.95	0.98	0.993
0.10	0.86	0.93	0.97	0.987
0.11	0.82	0.90	0.95	0.98
0.12	0.78	0.87	0.93	0.96
0.13	0.74	0.83	0.90	0.95
0.14	0.70	0.79	0.87	0.92
0.15	0.66	0.75	0.83	0.90
0.16	0.62	0.71	0.79	0.86
0.18	0.54	0.62	0.71	0.79
0.20	0.47	0.54	0.62	0.70
0.22	0.40	0.46	0.53	0.61
0.24	0.35	0.38	0.44	0.51
0.26	0.29	0.32	0.36	0.42
0.28	0.24	0.26	0.29	0.33
0.30	0.20	0.21	0.23	0.26
0.32	0.16	0.16	0.18	0.20
0.35	0.11	0.11	0.11	0.13
0.38	0.068	0.07	0.07	0.08
0.40	0.049	0.05	0.05	0.05
0.44	0.022	0.02	0.02	0.02
0.46	0.015	0.015	0.015	0.015
0.48	0.01	0.01	0.01	0.01
0.50	0.006	0.006	0.006	0.006

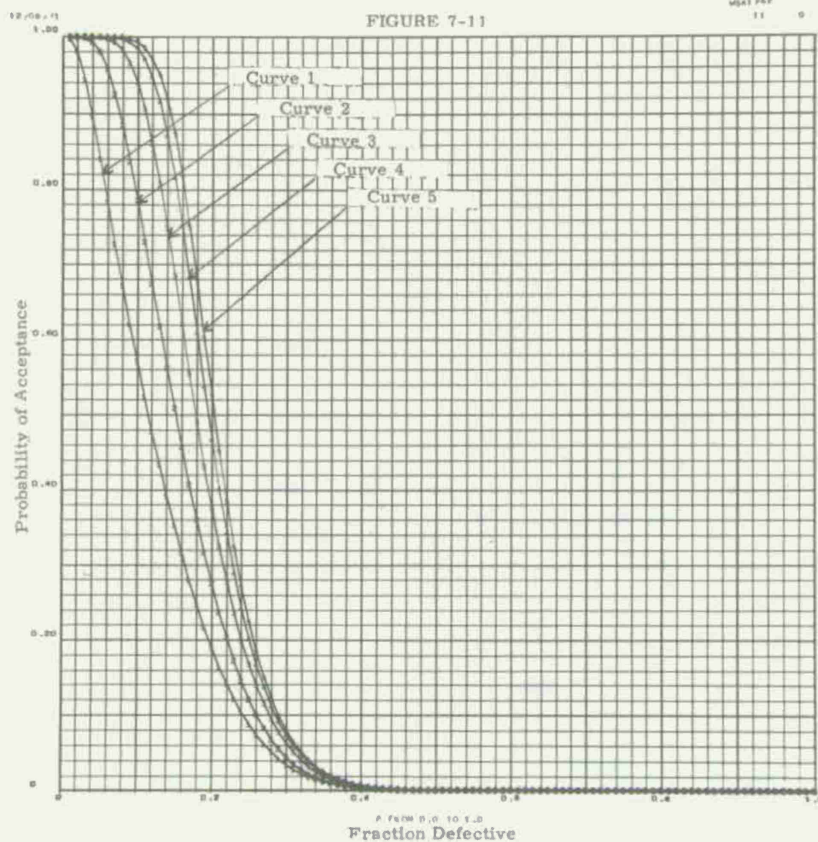


CHAIN SAMPLING PLAN DATA  
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TABLE 7-10

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=1 \ C_1=1$	Curve 2 $K_1=1 \ C_1=1$	Curve 3 $K_1=1 \ C_1=1$	Curve 4 $K_1=1 \ C_1=1$	Curve 5 $K_1=1 \ C_1=1$
	$K_2=3 \ C_2=2$	$K_2=3 \ C_2=5$	$K_2=3 \ C_2=8$	$K_2=3 \ C_2=12$	$K_2=3 \ C_2=10$
0.01	0.99	1.0	1.0	1.0	1.0
0.02	0.94	0.999	1.0	1.0	1.0
0.03	0.87	0.99	1.0	1.0	1.0
0.04	0.799	0.98	0.999	1.0	1.0
0.05	0.72	0.94	0.997	1.0	1.0
0.06	0.65	0.895	0.99	1.0	0.999
0.07	0.58	0.83	0.97	1.0	0.996
0.08	0.51	0.76	0.95	0.999	0.99
0.09	0.45	0.69	0.90	0.996	0.97
0.10	0.395	0.61	0.84	0.99	0.95
0.11	0.34	0.54	0.77	0.97	0.90
0.12	0.29	0.47	0.68	0.95	0.84
0.13	0.25	0.40	0.59	0.90	0.76
0.14	0.21	0.34	0.51	0.84	0.67
0.15	0.18	0.29	0.43	0.75	0.57
0.16	0.15	0.24	0.35	0.65	0.47
0.17	0.13	0.20	0.29	0.54	0.39
0.18	0.10	0.16	0.24	0.44	0.31
0.19	0.086	0.13	0.19	0.35	0.25
0.20	0.070	0.11	0.16	0.27	0.199
0.21	0.058	0.087	0.12	0.21	0.16
0.22	0.047	0.0698	0.0996	0.16	0.12
0.23	0.038	0.055	0.079	0.12	0.097
0.24	0.031	0.044	0.063	0.095	0.076
0.25	0.025	0.034	0.049	0.073	0.059
0.26	0.0196	0.027	0.039	0.056	0.046
0.27	0.016	0.021	0.030	0.043	0.036
0.28	0.012	0.016	0.023	0.033	0.028

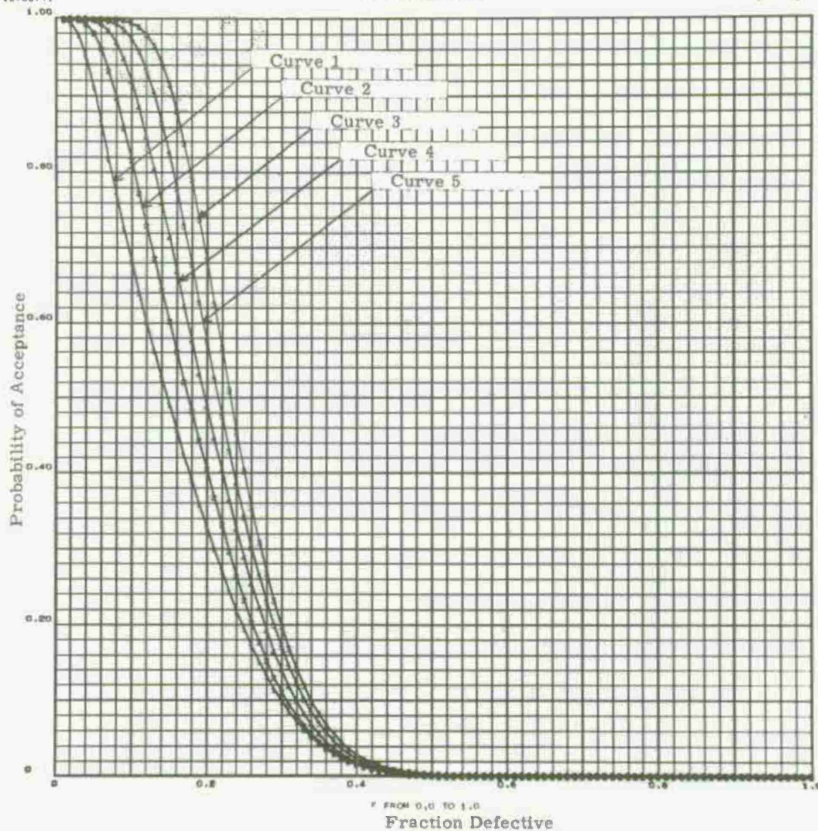




CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-11

Fraction Defective P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=2$ $K_2=3 \ C_2=3$	$K_1=1 \ C_1=2$ $K_2=3 \ C_2=6$	$K_1=1 \ C_1=2$ $K_2=3 \ C_2=9$	$K_1=1 \ C_1=2$ $K_2=3 \ C_2=11$	$K_1=1 \ C_1=2$ $K_2=3 \ C_2=12$
0.01	0.998	1.0	1.0	1.0	1.0
0.02	0.98	1.0	1.0	1.0	1.0
0.03	0.94	0.998	1.0	1.0	1.0
0.04	0.896	0.993	1.0	1.0	1.0
0.05	0.84	0.98	0.999	1.0	1.0
0.06	0.78	0.96	0.997	1.0	1.0
0.07	0.73	0.93	0.99	0.999	1.0
0.08	0.67	0.88	0.98	0.997	0.999
0.09	0.62	0.84	0.97	0.99	0.998
0.10	0.57	0.79	0.94	0.99	0.99
0.11	0.52	0.73	0.91	0.97	0.99
0.12	0.48	0.67	0.86	0.95	0.97
0.13	0.43	0.62	0.81	0.91	0.95
0.14	0.39	0.56	0.75	0.87	0.92
0.15	0.35	0.51	0.68	0.81	0.87
0.16	0.32	0.46	0.62	0.75	0.82
0.17	0.28	0.41	0.55	0.68	0.75
0.18	0.25	0.36	0.49	0.61	0.68
0.20	0.19	0.28	0.37	0.47	0.52
0.22	0.14	0.20	0.28	0.34	0.38
0.24	0.10	0.14	0.199	0.24	0.27
0.26	0.073	0.0998	0.14	0.17	0.18
0.28	0.051	0.067	0.094	0.11	0.12
0.30	0.035	0.044	0.062	0.073	0.079
0.35	0.012	0.014	0.0196	0.023	0.025
0.39	0.0046	0.00498	0.0069	0.0084	0.0090



CHAIN SAMPLING PLAN DATA  
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TABLE 7-12

Fraction Defective  P	Probability of Acceptance				
	Curve 1	Curve 2	Curve 3	Curve 4	Curve 5
	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=4$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=6$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=12$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=8$	$K_1=1 \ C_1=3$ $K_2=3 \ C_2=10$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.995	1.0	1.0	1.0	1.0
0.03	0.98	0.998	1.0	1.0	1.0
0.04	0.95	0.99	1.0	1.0	1.0
0.05	0.91	0.98	1.0	0.998	1.0
0.06	0.87	0.96	1.0	0.993	0.999
0.07	0.82	0.93	1.0	0.98	0.998
0.08	0.77	0.898	0.999	0.97	0.99
0.09	0.72	0.86	0.998	0.95	0.99
0.10	0.68	0.82	0.995	0.92	0.98
0.11	0.64	0.77	0.99	0.88	0.96
0.12	0.60	0.73	0.98	0.85	0.93
0.13	0.56	0.69	0.97	0.80	0.90
0.14	0.53	0.64	0.94	0.76	0.87
0.15	0.49	0.60	0.92	0.71	0.82
0.16	0.46	0.56	0.88	0.67	0.78
0.17	0.42	0.52	0.84	0.62	0.73
0.18	0.39	0.48	0.79	0.58	0.68
0.20	0.33	0.40	0.68	0.49	0.58
0.22	0.27	0.33	0.57	0.40	0.48
0.24	0.22	0.26	0.45	0.32	0.38
0.26	0.17	0.20	0.35	0.25	0.30
0.28	0.13	0.15	0.27	0.19	0.23
0.30	0.099	0.11	0.197	0.14	0.17
0.32	0.072	0.079	0.14	0.098	0.12
0.34	0.051	0.055	0.098	0.067	0.084
0.37	0.029	0.031	0.054	0.036	0.045
0.40	0.016	0.016	0.028	0.018	0.023
0.45	0.0049	0.00495	0.00797	0.0053	0.0063

FIGURE 7-13

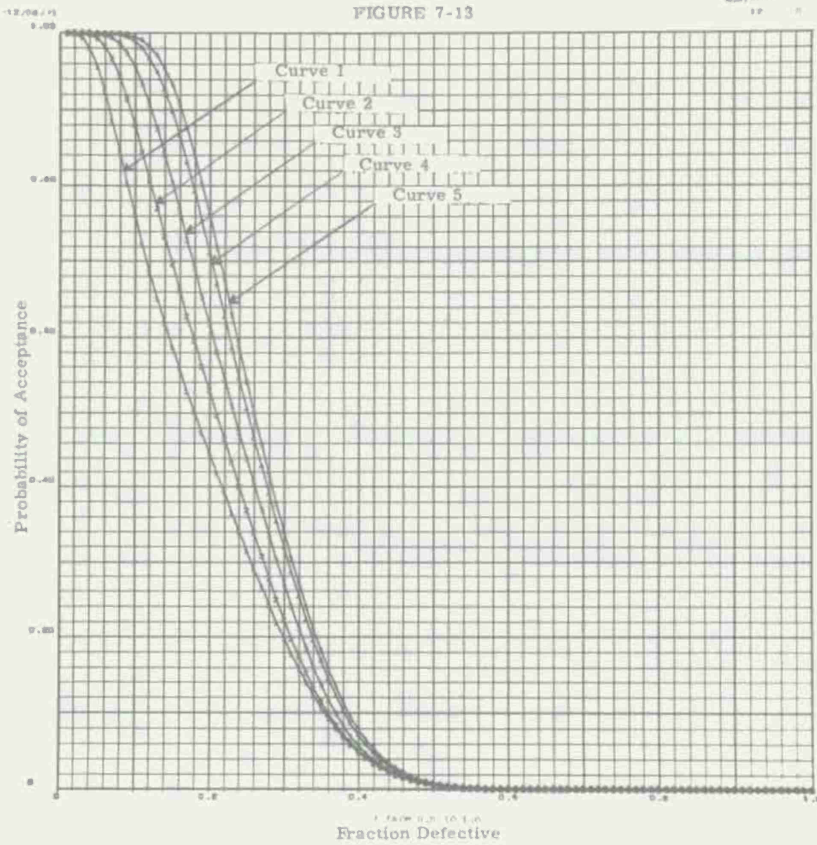
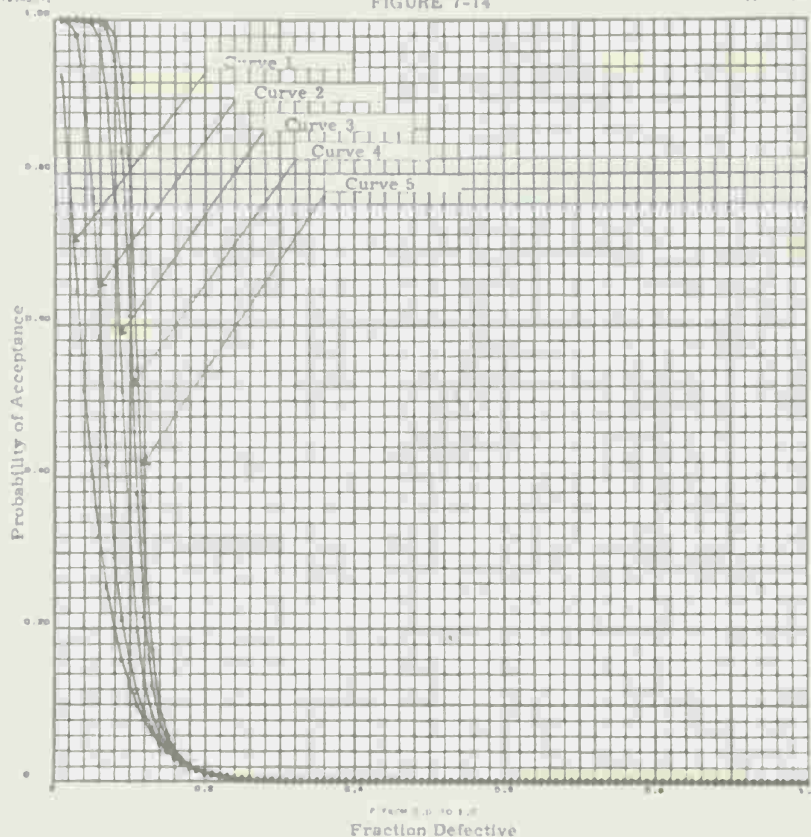
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-13

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=1$ $C_1=4$ $K_2=3$ $C_2=5$	Curve 2 $K_1=1$ $C_1=4$ $K_2=3$ $C_2=7$	Curve 3 $K_1=1$ $C_1=4$ $K_2=3$ $C_2=9$	Curve 4 $K_1=1$ $C_1=4$ $K_2=3$ $C_2=11$	Curve 5 $K_1=1$ $C_1=4$ $K_2=3$ $C_2=12$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.999	1.0	1.0	1.0	1.0
0.03	0.993	1.0	1.0	1.0	1.0
0.04	0.98	0.998	1.0	1.0	1.0
0.05	0.96	0.99	0.999	1.0	1.0
0.06	0.92	0.98	0.998	1.0	1.0
0.07	0.89	0.97	0.99	0.999	1.0
0.08	0.84	0.94	0.99	0.998	0.999
0.09	0.80	0.91	0.97	0.99	0.998
0.10	0.76	0.88	0.96	0.99	0.995
0.11	0.72	0.84	0.93	0.98	0.99
0.12	0.69	0.81	0.91	0.97	0.98
0.13	0.65	0.77	0.87	0.95	0.97
0.14	0.62	0.73	0.84	0.92	0.95
0.16	0.56	0.66	0.77	0.86	0.91
0.18	0.50	0.59	0.69	0.79	0.84
0.20	0.45	0.53	0.62	0.71	0.76
0.22	0.39	0.47	0.54	0.63	0.67
0.24	0.34	0.40	0.47	0.54	0.58
0.26	0.29	0.34	0.40	0.46	0.498
0.28	0.24	0.28	0.34	0.39	0.42
0.30	0.197	0.22	0.27	0.32	0.34
0.32	0.16	0.17	0.21	0.25	0.27
0.34	0.12	0.13	0.16	0.19	0.21
0.37	0.0796	0.084	0.098	0.12	0.13
0.40	0.049	0.050	0.057	0.071	0.078
0.43	0.028	0.028	0.031	0.038	0.042
0.46	0.015	0.015	0.016	0.019	0.021
0.49	0.0075	0.0075	0.0078	0.0089	0.0099



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TABLE 7-14

Fraction Defective  P	Probability of Acceptance				
	Curve 1 $K_1=2 C_1=0$	Curve 2 $K_1=2 C_1=0$	Curve 3 $K_1=2 C_1=0$	Curve 4 $K_1=2 C_1=0$	Curve 5 $K_1=2 C_1=0$
	$K_2=3 C_2=1$	$K_2=3 C_2=5$	$K_2=3 C_2=9$	$K_2=3 C_2=11$	$K_2=3 C_2=12$
0.01	0.93	1.0	1.0	1.0	1.0
0.02	0.79	0.998	1.0	1.0	1.0
0.03	0.64	0.98	1.0	1.0	1.0
0.04	0.51	0.91	1.0	1.0	1.0
0.05	0.41	0.77	0.995	1.0	1.0
0.06	0.32	0.58	0.97	0.99	1.0
0.07	0.25	0.42	0.90	0.99	0.997
0.08	0.20	0.29	0.74	0.95	0.98
0.09	0.16	0.21	0.51	0.83	0.93
0.10	0.13	0.16	0.32	0.61	0.78
0.11	0.0995	0.12	0.197	0.38	0.54
0.12	0.070	0.088	0.13	0.21	0.31
0.13	0.062	0.068	0.086	0.12	0.17
0.14	0.049	0.053	0.061	0.078	0.097
0.15	0.039	0.041	0.045	0.052	0.060
0.16	0.031	0.032	0.034	0.037	0.041
0.17	0.024	0.025	0.026	0.027	0.029
0.18	0.019	0.019	0.0198	0.021	0.021
0.19	0.015	0.015	0.015	0.016	0.016
0.20	0.012	0.012	0.012	0.012	0.012
0.21	0.0097	0.0090	0.0091	0.0092	0.0093
0.25	0.0032	0.0032	0.0032	0.0032	0.0032
0.30	0.000798	0.000798	0.000799	0.000799	0.000799
0.35	0.00018	0.00018	0.00018	0.00018	0.00018
0.40	0.000037	0.000037	0.000037	0.000037	0.000037
0.45					0.0000064



FIGURE 7-15

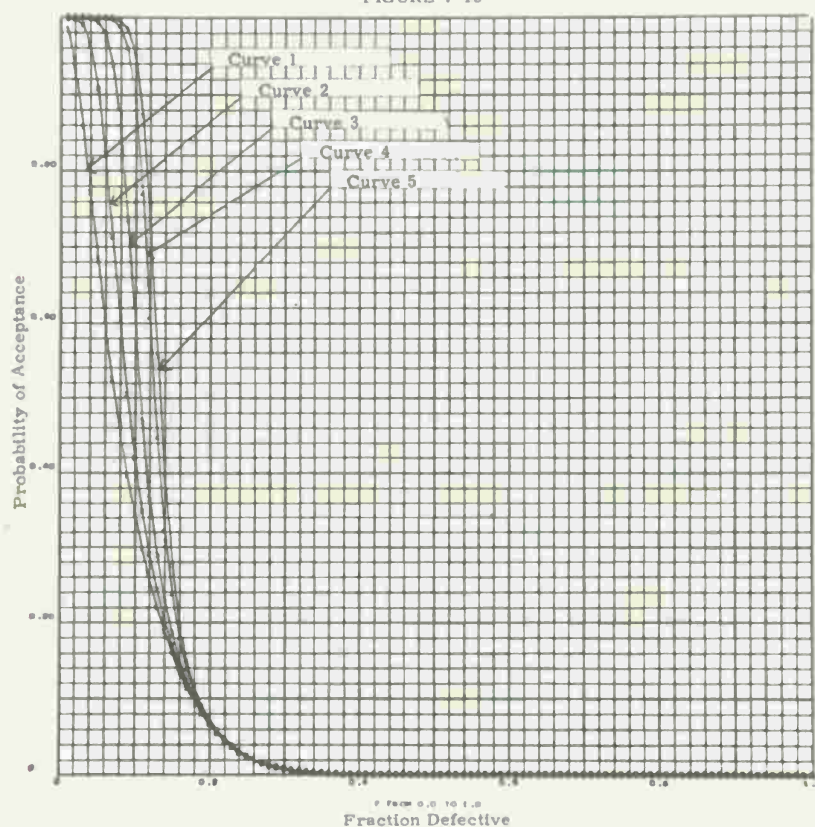
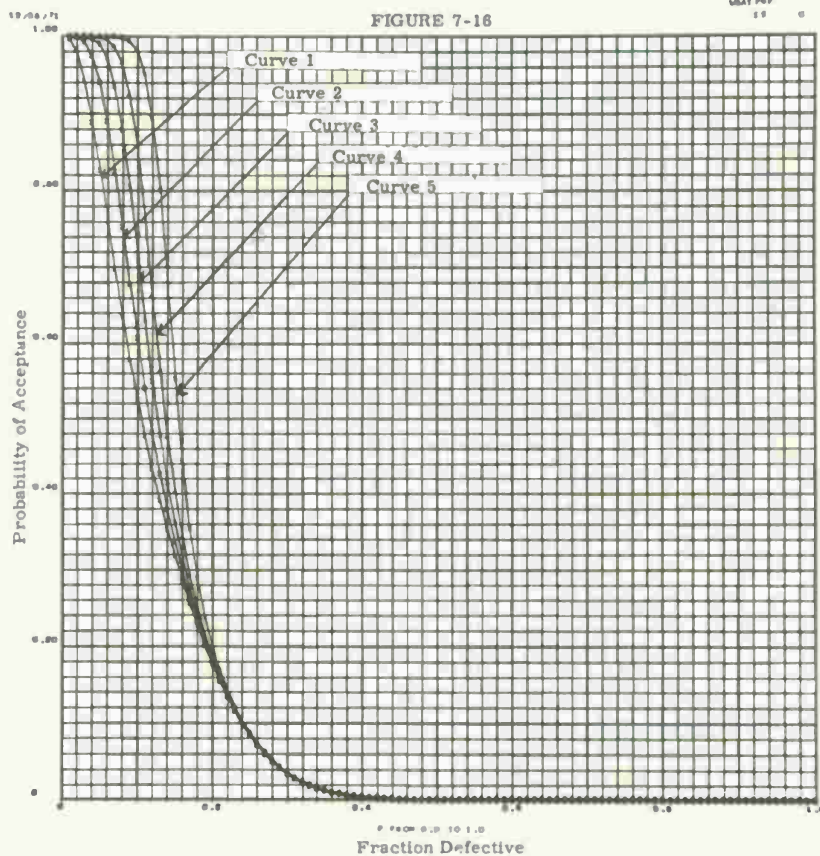
CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-15

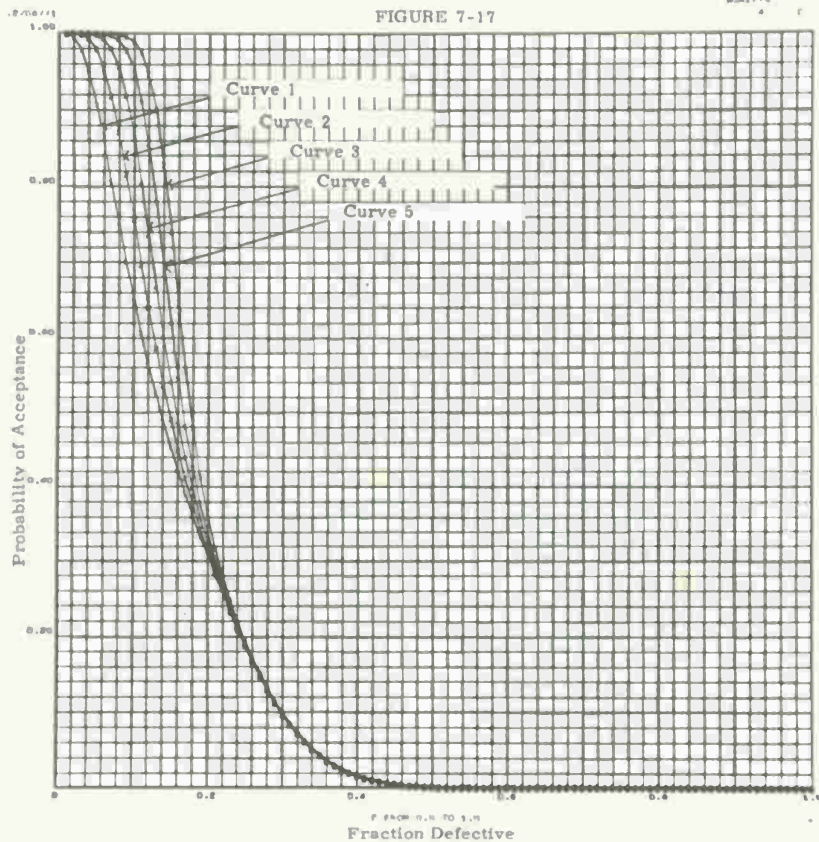
Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=2 C_1=1$ $K_2=3 C_2=2$	Curve 2 $K_1=2 C_1=1$ $K_2=3 C_2=5$	Curve 3 $K_1=2 C_1=1$ $K_2=3 C_2=8$	Curve 4 $K_1=2 C_1=1$ $K_2=3 C_2=11$	Curve 5 $K_1=2 C_1=1$ $K_2=3 C_2=12$
0.01	0.99	1.0	1.0	1.0	1.0
0.02	0.93	0.999	1.0	1.0	1.0
0.03	0.86	0.99	1.0	1.0	1.0
0.04	0.77	0.96	0.999	1.0	1.0
0.05	0.68	0.91	0.995	1.0	1.0
0.06	0.60	0.82	0.98	0.999	1.0
0.07	0.53	0.71	0.94	0.997	0.999
0.08	0.46	0.60	0.86	0.99	0.996
0.09	0.398	0.50	0.73	0.96	0.98
0.10	0.34	0.42	0.596	0.89	0.95
0.11	0.298	0.35	0.47	0.77	0.87
0.12	0.26	0.29	0.37	0.61	0.73
0.13	0.22	0.24	0.29	0.45	0.55
0.14	0.19	0.20	0.23	0.32	0.39
0.15	0.16	0.17	0.19	0.24	0.28
0.16	0.14	0.14	0.15	0.18	0.199
0.17	0.11	0.12	0.12	0.14	0.15
0.18	0.096	0.098	0.10	0.11	0.11
0.19	0.08	0.081	0.083	0.087	0.09
0.20	0.066	0.067	0.068	0.070	0.072
0.21	0.054	0.055	0.056	0.057	0.058
0.22	0.045	0.045	0.045	0.046	0.046
0.23	0.036	0.037	0.037	0.037	0.037
0.24	0.03	0.03	0.03	0.03	0.03
0.25	0.024	0.024	0.024	0.024	0.024
0.26	0.019	0.019	0.019	0.019	0.019
0.27	0.015	0.015	0.015	0.015	0.015
0.28	0.012	0.012	0.012	0.012	0.012
0.29	0.0096	0.0096	0.0097	0.0097	0.0097
0.30	0.0076	0.0076	0.0076		



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TABLE 7-16

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=2$ $C_1=2$	Curve 2 $K_1=2$ $C_1=2$	Curve 3 $K_1=2$ $C_1=2$	Curve 4 $K_1=2$ $C_1=2$	Curve 5 $K_1=2$ $C_1=2$
	$K_2=3$ $C_2=3$	$K_2=3$ $C_2=5$	$K_2=3$ $C_2=7$	$K_2=3$ $C_2=9$	$K_2=3$ $C_2=12$
0.01	0.998	1.0	1.0	1.0	1.0
0.02	0.98	0.999	1.0	1.0	1.0
0.03	0.94	0.99	1.0	1.0	1.0
0.04	0.89	0.98	0.997	1.0	1.0
0.05	0.83	0.94	0.99	0.999	1.0
0.06	0.76	0.89	0.97	0.996	1.0
0.07	0.70	0.83	0.94	0.99	1.0
0.08	0.64	0.75	0.88	0.97	0.998
0.09	0.58	0.68	0.80	0.92	0.99
0.10	0.53	0.61	0.72	0.86	0.98
0.12	0.43	0.48	0.55	0.66	0.89
0.14	0.35	0.38	0.41	0.47	0.68
0.16	0.29	0.299	0.32	0.34	0.44
0.18	0.23	0.23	0.24	0.25	0.29
0.20	0.18	0.18	0.18	0.19	0.199
0.22	0.13	0.14	0.14	0.14	0.14
0.24	0.099	0.0996	0.10	0.10	0.10
0.27	0.060	0.060	0.060	0.061	0.061
0.30	0.034	0.034	0.034	0.034	0.035
0.33	0.019	0.019	0.019	0.019	0.019
0.36	0.0095	0.0095	0.0095	0.0095	0.0095

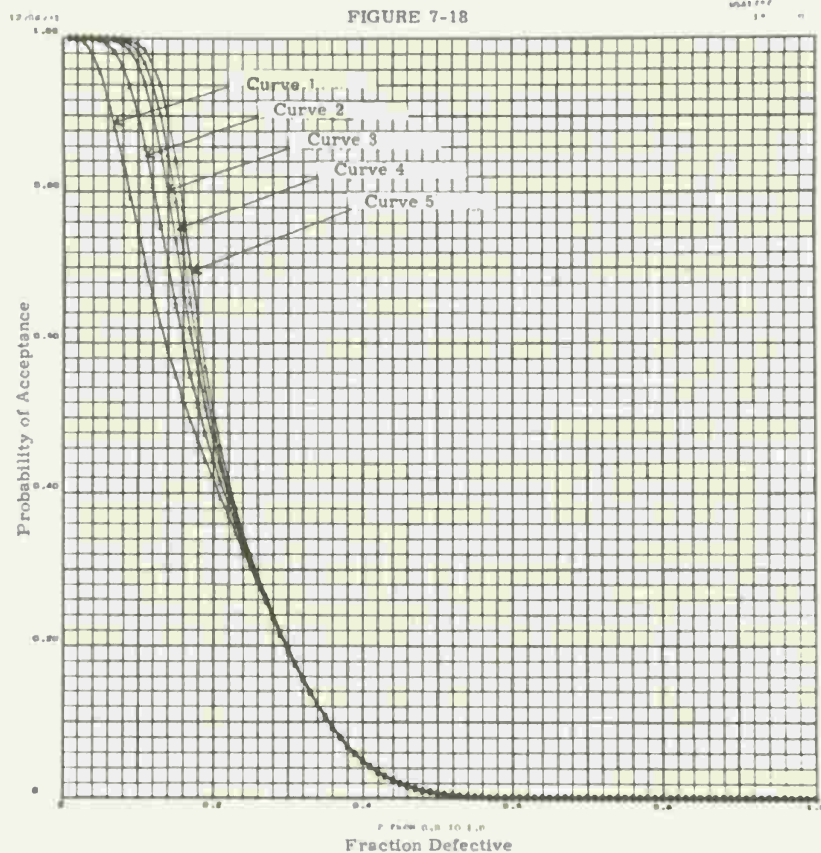


CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-17

Fraction Defective P	Probability of Acceptance				
	Curve 1 $K_1=2$ $C_1=3$ $K_2=3$ $C_2=4$	Curve 2 $K_1=2$ $C_1=3$ $K_2=3$ $C_2=6$	Curve 3 $K_1=2$ $C_1=3$ $K_2=3$ $C_2=12$	Curve 4 $K_1=2$ $C_1=3$ $K_2=3$ $C_2=8$	Curve 5 $K_1=2$ $C_1=3$ $K_2=3$ $C_2=10$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.995	1.0	1.0	1.0	1.0
0.03	0.98	0.998	1.0	1.0	1.0
0.04	0.95	0.99	1.0	1.0	1.0
0.05	0.91	0.98	1.0	0.997	1.0
0.06	0.86	0.95	1.0	0.99	0.999
0.07	0.81	0.92	1.0	0.98	0.997
0.08	0.75	0.87	0.999	0.96	0.99
0.09	0.70	0.82	0.996	0.92	0.98
0.10	0.65	0.76	0.99	0.87	0.95
0.11	0.61	0.698	0.98	0.80	0.91
0.12	0.56	0.64	0.95	0.74	0.85
0.13	0.52	0.59	0.899	0.67	0.78
0.14	0.49	0.54	0.83	0.60	0.698
0.17	0.39	0.41	0.56	0.44	0.48
0.20	0.31	0.31	0.36	0.32	0.34
0.24	0.21	0.21	0.22	0.21	0.22
0.28	0.13	0.13	0.13	0.13	0.13
0.30	0.097	0.097	0.098	0.097	0.098
0.34	0.051	0.051	0.051	0.051	0.051
0.38	0.024	0.024	0.024	0.024	0.024
0.40	0.016	0.016	0.016	0.016	0.016
0.43	0.00798	0.00798	0.00798	0.00798	0.00798





CHAIN SAMPLING PLAN DATA  
SAMPLE SIZE 20

TABLE 7-18

Fraction Defective  P	Probability of Acceptance				
	Curve 1 $K_1=2 \ C_1=4$	Curve 2 $K_1=2 \ C_1=4$	Curve 3 $K_1=2 \ C_1=4$	Curve 4 $K_1=2 \ C_1=4$	Curve 5 $K_1=2 \ C_1=4$
	$K_2=3 \ C_2=5$	$K_2=3 \ C_2=8$	$K_2=3 \ C_2=10$	$K_2=3 \ C_2=11$	$K_2=3 \ C_2=12$
0.01	1.0	1.0	1.0	1.0	1.0
0.02	0.999	1.0	1.0	1.0	1.0
0.03	0.994	1.0	1.0	1.0	1.0
0.04	0.98	1.0	1.0	1.0	1.0
0.05	0.96	0.998	1.0	1.0	1.0
0.06	0.92	0.99	0.999	1.0	1.0
0.07	0.88	0.98	0.997	0.999	1.0
0.08	0.84	0.96	0.993	0.997	0.999
0.09	0.79	0.94	0.98	0.99	0.997
0.10	0.75	0.90	0.97	0.98	0.99
0.11	0.70	0.86	0.94	0.97	0.98
0.12	0.66	0.81	0.90	0.94	0.97
0.14	0.59	0.70	0.796	0.85	0.897
0.20	0.42	0.44	0.47	0.49	0.51
0.24	0.32	0.33	0.34	0.34	0.35
0.28	0.24	0.24	0.24	0.24	0.24
0.32	0.16	0.16	0.16	0.16	0.16
0.36	0.092	0.092	0.092	0.092	0.092
0.38	0.068	0.068	0.068	0.068	0.068
0.40	0.049	0.049	0.049	0.049	0.049
0.42	0.034	0.034	0.034	0.034	0.034
0.44	0.023	0.023	0.023	0.023	0.023
0.46	0.015	0.015	0.015	0.015	0.015
0.50	0.0059	0.0059	0.0059	0.0059	0.0059

## APPENDIX A

### MATHEMATICAL PROCEDURE

The quantities needed in chain sampling are:

1. "n" - the size sample drawn from each lot
2.  $K_1$  - the number of lots for the initial and restart stage

(hereinafter called restart stage) of sampling

3.  $c_1$  - the allowable number of failures in the samples from the  $K_1$  lots

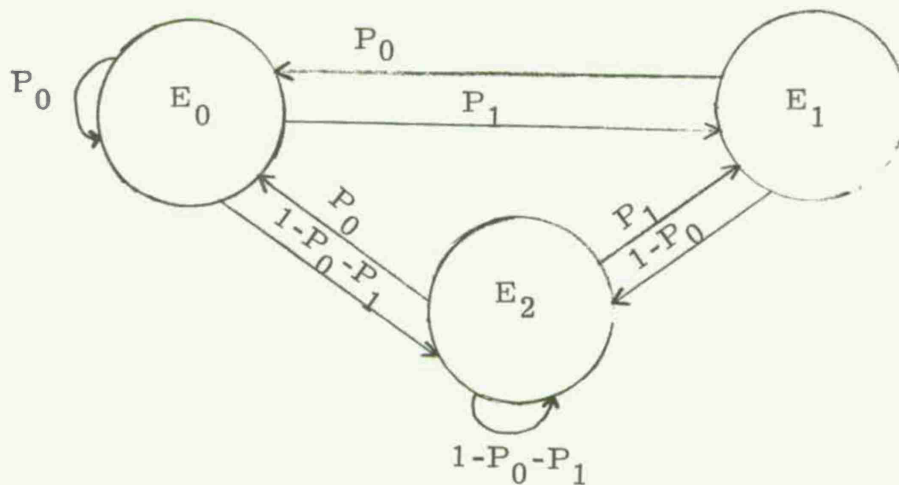
4.  $K_2$  - the number of lots for the normal operating stage of sampling, where  $K_2$  is greater than  $K_1$

5.  $c_2$  - the allowable number of failures in the samples from the  $K_2$  lots, where  $c_2$  is greater than  $c_1$

A Markov model is very convenient for computation of the operating characteristic (OC) curve for chain sampling given the quantities  $n$ ,  $K_1$ ,  $c_1$ ,  $K_2$ , and  $c_2$ . Although the sample size does not appear as such in any of the matrices, probability values in the matrices are based on sample size. If sample sizes differ for  $K_1$  and  $K_2$ , subscripts must be used on the probability values in the transition matrix.

To illustrate the validity of the Markov model, let the plan to be considered be  $(K_1, K_2; c_1, c_2) = (0, 2; 0, 1)$ . There are three event-states

to be considered,  $E_0$ ,  $E_1$ , and  $E_2$  where  $E_0$  is an acceptance event, no defectives;  $E_1$  is an acceptance event, one defective;  $E_2$  is event of lot rejection. The following diagram shows the transition probabilities between the event-states,  $E_j$  for  $j=0, 1, 2$ .



Notice that the sum of the probabilities emanating from each  $E_j$  is unity. For example, the arrows having their source in  $E_0$  have as their sum  $P_0 + P_1 + (1-P_0-P_1) = 1$ . Note: This property yields a stochastic transition matrix, i. e., a matrix, each row of which adds up to one.

The requirements of the plan preclude the possibility of going from  $E_1$  back to  $E_1$ ; because to be in  $E_1$  one defective has occurred and the occurrence of another will sum to two, which exceeds the maximum number of defectives allowed by the plan.

The transition matrix can then be set up as follows:

		ith state		
		$E_0$	$E_1$	$E_2$
(i-1)th state	$E_0$	$P_0$	$P_1$	$1-P_0-P_1=P_R$
	$E_1$	$P_0$	0	$1-P_0=P'_R$
	$E_2$	$P_0$	$P_1$	$1-P_0-P_1=P''_R$

If now the equations are set up for the probabilities of each event-state on the  $i$ th trial, the results are equations 1, 2, 3; equation 4 is known before hand and must be used.

$$\begin{array}{ll}
 \text{A} & \begin{array}{l}
 1. \quad p_{0(i)} = p_{0(i-1)} P_0 + p_{1(i-1)} P_0 + p_{R(i-1)} P_0 \\
 2. \quad p_{1(i)} = p_{0(i-1)} P_1 + p_{R(i-1)} P_1 \\
 3. \quad p_{R(i)} = p_{0(i-1)} P_R + p_{1(i-1)} P'_R + p_{R(i-1)} P''_R \\
 4. \quad 1 = p_{0(i-1)} + p_{1(i-1)} + p_{R(i-1)}
 \end{array}
 \end{array}$$

There are now four equations in three unknowns, since  $p_{0i}$ ,  $p_{1i}$ , and  $p_{Ri}$  are exactly equal to  $p_{0(i-1)}$ ,  $p_{1(i-1)}$ , and  $p_{R(i-1)}$  in the steady state. Utilizing equations 4, 1, and 2, they are solved simultaneously for  $p_{Ri}$ .

The equations are:

$$\begin{array}{ll}
 \text{B} & \begin{array}{l}
 1. \quad p_0 + p_1 + p_R = 1 \\
 2. \quad (P_0-1)p_0 + P_0p_1 + P_0p_R = 0 \\
 3. \quad P_1p_0 - p_1 + P_1p_R = 0
 \end{array}
 \end{array}$$

Solving by matrix methods gives:

$$\begin{pmatrix} 1 & 1 & 1 \\ P_0^{-1} & P_0 & P_0 \\ P_1 & -1 & P_1 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_R \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{or}$$

A  $p=b$  then  $p=A^{-1}b$ , where A is a three by three matrix and p is a three by one matrix.

From this set of equations  $p_R$ , the probability of rejection, and  $p_A=(1-p_R)$ , the probability of acceptance, are determined for any sample size n and process fraction defective p by substituting the appropriate values of  $P_0$  and  $P_1$  from the binomial expansion into the coefficient matrix A. ( $P_0$  is the probability of zero failures and  $P_1$  the probability of one failure in n trials of an event having probability p' of occurring on any trial.)

Any plan ( $K_1, K_2; c_1, c_2$ ), with equal sample sizes in the two stages, will hereinafter be written as an identification matrix. The example plan (page <sup>67</sup>105) is (0, 2; 0, 1) with

$$\text{Identification matrix A} = \begin{matrix} & \begin{pmatrix} K & c \\ 0 & 0 \\ 2 & 1 \end{pmatrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \end{matrix}$$

1. Row one defines the restart stage.
2. Row two defines the normal operating stage.

3. Column one denotes the number of lots in the chain in the two stages.

4. Column two denotes the allowable number of defectives in the associated  $K$  samples from the  $K$  lots for the two stages.

In identification matrix  $A$ , there is no  $K_1$  and no  $c_1$  ( $K_1=0$ ,  $c_1=0$ ).

It is a simple chain wherein the current lot will be accepted as long as the number of defectives in the sample from the last lot plus the number of defectives in the sample from the current lot is less than or equal to one.

A state defining matrix  $B$  and a transition matrix  $C$  may be set up.

		state at $i$ th trial			
		0	1	R	
$B=$	$(i-1)$ th state	0	00	01	0R
	1	10			1R
	R	R0	R1		RR

Possible states for two samples when no more than one defective can be permitted

		state at $i$ th trial			
		0	1	R	
$C =$	$(i-1)$ th state	0	$P_0$	$P_1$	$P_{0R}$
	1	$P_0$			$P_{1R}$
	R	$P_0$	$P_1$		$P_{RR}$

Probabilities of going from  $(i-1)$ th state to the  $i$ th state.

Note: This is a stochastic matrix because the sum of the row elements add up to unity.



To consider a two-stage plan (with a restart as well as a normal stage), the identification matrix  $A$  is now changed by putting  $K_1=1$  and  $c_1=0$  in identification matrix  $D$  as shown.

$$D = \begin{matrix} & \begin{matrix} K & c \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \end{matrix}$$

This corresponds to tightening inspection after a lot is rejected and at the start of the process. At the start of the process a single sample, from the first lot, must pass the test with no defectives. After the first lot; however, a single defective in a chain of two samples will be acceptable without rejecting a lot. The process is as previously described except that when the rejection of a lot occurs, the restart procedure is initiated whereas before the normal operating stage was restarted.

The state defining matrix  $E$  associated with the identification matrix  $D$  is:

$$E = \begin{array}{c|ccc} & 0 & 1 & R \\ \hline 0 & 00 & 01 & 0R \\ \hline 1 & 10 & & 1R \\ \hline R & R0 & & RR \end{array}$$



A comparison of matrix E with matrix B (page <sup>76</sup>~~109~~) reveals the effect of the  $K_1$  samples with a  $c_1=0$ . Sequence R1, which appears in matrix B, is dropped from matrix E since it no longer represents an acceptable sequence. The requirement,  $c_1=0$ , of the restart stage precludes any acceptable sequence other than R0. Again the state defining matrix E and the transition matrix F are the same.

F =

	0	1	R
0	$P_0$	$P_1$	$P_{0R}$
1	$P_0$		$P_{1R}$
R	$P_0$		$P_{RR}$

Writing the equations yields a set of equations C.

$$\begin{array}{lcl}
 \text{C} & \begin{array}{l} 1. \\ 2. \\ 3. \\ 4. \end{array} & \begin{array}{l} p_{0i} = P_0 p_{0(i-1)} + P_0 p_{1(i-1)} + P_0 p_{R(i-1)} \\ p_{1i} = P_1 p_{0(i-1)} \\ p_{Ri} = P_R p_{0(i-1)} + P_R p_{1(i-1)} + P_R p_{R(i-1)} \\ 1 = p_{0(i-1)} + p_{1(i-1)} + p_{R(i-1)} \end{array}
 \end{array}$$

A comparison of the set of equations C with the set of equations A (page <sup>69</sup>~~107~~) reveals the effect of the restart procedure on the equations themselves. Equation 4 is known as usual, and the usual matrix methods may be applied to the solution of the simultaneous equations.

The matrices previously shown are simple, but small increases in the magnitude of the parameters rapidly increases the complexity of the system of equations. For example, suppose an identification matrix G is:

$$G = \begin{matrix} & \begin{pmatrix} K & c \\ 1 & 1 \\ 2 & 2 \end{pmatrix} \end{matrix}$$

All possible transition sequences are depicted in the state-defining matrix H (page <sup>75</sup>~~143~~). Normal stage sequences are represented by three-digit symbols ( $K_2=3$ ), the digits 0, 1, 2 standing for the number of defectives observed in accepted lots and R identifying rejected lots. Thus, 000, 001, 002, 010, 011, 020, 100, 101, 110, 200, R00, R01, R02, R10, and R11 represent acceptable sequences. It is noted that the sum of the digits is always equal to or less than  $c_2=2$  not counting rejected lots. Restart stage sequences are represented by an R followed by a single ( $K_1=1$ ) digit 0 or 1 for acceptable sequences or R for an unacceptable sequence. It is noted that the restart sequence (lot) must be equal to or less than  $c_1$  to be acceptable. Since in going from one state to the next the oldest lot result is dropped, the states may be completely identified by the last two digits of the sequence leading to the given state, except the reject state, which is represented by the single symbol R.

H=

	00	01	02	10	11	20	R0	R1	R
00	000	001	002						00R
01				010	011				01R
02						020			02R
10	100	101							10R
11				110					11R
20	200								20R
R0	R00	R01	R02						R0R
R1				R10	R11				R1R
R							R0	R1	RR

The transition matrix  $I$  may be easily obtained from the state defining matrix by substituting  $P_j$  for accept sequences where  $j=0, 1, 2$  is the last digit of the sequence and one minus the sum of the row probabilities for the reject sequences.  $P_j$  as before is the binomial probability of obtaining  $j$  defectives out of a sample of  $n$  units given the single-draw probability  $p$ . The steady-state solution for the state probabilities is then obtained as described previously. It is noted that the reject probabilities are not used in the steady state solution and, therefore, need not be calculated.

	00	01	02	10	11	20	R0	R1	R
00	$P_0$	$P_1$	$P_2$						$1 - \sum$
01				$P_0$	$P_1$				$1 - \sum$
02						$P_0$			$1 - \sum$
10	$P_0$	$P_1$							$1 - \sum$
11				$P_0$					$1 - \sum$
20	$P_0$								$1 - \sum$
R0	$P_0$	$P_1$	$P_2$						$1 - \sum$
R1				$P_0$	$P_1$				$1 - \sum$
R							$P_0$	$P_1$	$1 - \sum$

In the above chain plan, the states R0 and R1 are intermediate states required to represent the buildup from  $K_1=1$  to  $K_2=3$ , i. e., the actual sequence governed by the criterion  $c_2=2$  is only two lots long since R is not counted. However, if we increase  $K_1=1$  to  $K_1=2$  in the above plan, the same states will be defined as before, except that transitions from R0 and R1 will now be governed by  $c_1=1$  instead of  $c_2=2$ .

Until now we have considered only equal sample sizes for restart and normal stages. It is, of course, possible and perhaps desirable to use larger sample sizes for restart procedures to take advantage of the better discrimination obtained and combine this with smaller sample sizes during the normal stage when the process is not suspect. It is possible by this

means to devise chain plans with similar O. C. curves as equal sample size plans, but with lower average sample sizes.

The identification matrix G (page ~~112~~<sup>74</sup>) may be altered to include the sample size information as follows:

$$J = \begin{matrix} & \begin{matrix} K & c & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} 1 & 1 & 10 \\ 3 & 2 & 5 \end{pmatrix} \end{matrix}$$

There are now  $n_1=10$  units per sample for stage 1 (restart) and  $n_2=5$  units per sample for stage 2 (normal).

The state-defining matrix will be the same as matrix H on page ~~113~~<sup>75</sup>. However, in setting up the transition matrix, the transition probabilities must be subscripted with identification of the appropriate sample size. Therefore, we may designate  $P_{j, n_1}$  and  $P_{j, n_2}$  to represent the binomial probabilities.

Examining the identification matrix J and the state-defining matrix H, it is readily noted that  $n_1$  applies only to the lot immediately following a rejection and thus only occurs in the transitions from R to R0 or R1. All other transition probabilities involve a sample size of  $n_2$ . The transition matrix may then be written as follows:

	00	01	02	10	11	20	R0	R1	R
00	$P_{0,n_2}$	$P_{1,n_2}$	$P_{2,n_2}$						$1-\xi$
01				$P_{0,n_2}$	$P_{1,n_2}$				$1-\xi$
02						$P_{0,n_2}$			$1-\xi$
10	$P_{0,n_2}$	$P_{1,n_2}$							$1-\xi$
11				$P_{0,n_2}$					$1-\xi$
20	$P_{0,n_2}$								$1-\xi$
R0	$P_{0,n_2}$	$P_{1,n_2}$	$P_{2,n_2}$						$1-\xi$
R1				$P_{0,n_2}$	$P_{1,n_2}$				$1-\xi$
R							$P_{0,n_1}$	$P_{1,n_1}$	$1-\xi$

The steady state solution for the state probabilities is then obtained as before.

With the introduction of differing sample sizes, the question of average sample size or number (ASN) arises. The ASN may be determined from steady state probabilities by utilizing the standard expected value formula,

$$ASN = n_1 p(n_1) + n_2 p(n_2),$$

where  $n_1$  and  $n_2$  are the sample sizes for restart and normal stages respectively and  $p(n_1)$  and  $p(n_2)$  are their steady state probabilities of occurrence. Since  $n_1$  and  $n_2$  are the only alternatives, the sum of their probabilities of occurrence is one.



$$p(n_1) + p(n_2) = 1$$

Therefore, it is necessary to determine only one of the two steady state probabilities.

The probability,  $p(n_1)$ , may be determined from the state probabilities using the following reasoning. The case of  $K_1=1$  will be considered first since it is simpler and more easily visualized. In this case each rejected lot is always followed by one lot of sample size  $n_1$ , and all other lots will have sample size  $n_2$ . By examining various combinations including consecutive rejections, it is discovered that a one-to-one correspondence can be set up between rejected lots and  $n_1$  lots. The number of lots with sample size  $n_1$  will then be equal to the number of rejected lots and their steady state probabilities will be equal.

$$p(n_1) = p_R = 1 - p_A$$

$$p(n_2) = 1 - p(n_1) = p_A$$

The ASN may then be calculated by substituting in the expected value equation as follows:

$$\begin{aligned} \text{ASN} &= n_1 p(n_1) + n_2 p(n_2) \\ &= n_1 (1 - p_A) + n_2 p_A \\ &= n_1 + (n_2 - n_1) p_A \end{aligned}$$

Since the chain plans previously described in this appendix are all  $K_1=1$  plans, the above formula would be applicable.

When  $K_1$  is greater than one, the reasoning becomes more complicated because of overlapping in the area of consecutive rejections. For instance, consider the following plan:

$$\begin{matrix} & \begin{pmatrix} K & c & n \\ 1 & 2 & 1 & n_1 \\ 2 & 3 & 2 & n_2 \end{pmatrix} \end{matrix}$$

The state defining matrix is then

	00	01	02	10	11	20	R0	R1	R
00	000	001	002						00R
01				010	011				01R
02						020			02R
10	100	101							10R
11				110					11R
20	200								20R
R0	R00	R01							R0R
R1				R10					R1R
R							R0	R1	RR

We then reason as follows. The lot immediately following a lot in states R, R0, and R1 always requires a sample size of  $n_1$  and all other cases will require  $n_2$ . Therefore, a one-to-one correspondence can be established between the above states and  $n_1$  lots. The number of lots with sample size  $n_1$  will then be equal to the number of rejected lots plus the number of R0 and R1 states. The probability of occurrence of  $n_1$  will then be equal to the sum of the probabilities of occurrence of R, R0, and R1 states.

$$p(n_1) = p_R + p_{R0} + p_{R1}$$

The relation between  $p_{R0}$  and  $p_{R1}$  and  $p_R$  may be obtained by generating the state equations for  $p_{R0}$  and  $p_{R1}$  from the transition matrix as follows:

$$p_{R0} = p_R \cdot P_0$$

$$p_{R1} = p_R \cdot P_1$$

It is noted that these formulas could also be determined directly from the product formula assuming independence. Substituting these results into the equation for  $p(n_1)$ , we get

$$\begin{aligned} p(n_1) &= p_R + p_{R0} + p_{R1} \\ &= p_R + P_0 p_R + P_1 p_R \\ &= p_R (1 + P_0 + P_1) \\ &= (1 - P_A) (1 + P_0 + P_1) \end{aligned}$$

Therefore,

$$\begin{aligned}\text{ASN} &= n_1 p(n_1) + n_2 p(n_2) \\ &= n_1 (1 - P_A) (1 + P_0 + P_1) + n_2 [1 - (1 - P_A) (1 + P_0 + P_1)] \\ &= n_2 + (n_1 - n_2) (1 - P_A) (1 + P_0 + P_1)\end{aligned}$$

This logic can be extended to the computation of an ASN curve for any chain sampling plan with differing sample sizes in the two stages.

## APPENDIX B

### COMPUTATION METHODS

The computation of operating characteristic (OC) curves and average sample number (ASN) curves for complex chain sampling plans is very laborious and almost impossible to do manually. In order to computerize the computation of OC and ASN curves, a scheme must be established to systematize the process of generating a state defining matrix, transition matrix, the equations defining the state probabilities, and to obtain the steady-state solution to these equations for the range of fraction defective that will define the OC curve. To illustrate the method that MICOM has developed, consider the following:

$$\text{Identification matrix } A = \begin{matrix} & \begin{matrix} K & c & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} 1 & 1 & n_1 \\ 3 & 2 & n_2 \end{pmatrix} \end{matrix}$$

and

State Defining Matrix B =	j k									
	ij	00	01	02	10	11	20	R0 (30)	R1 (31)	R (32)
	00	000	001	002						
	01				010	011				
	02						020			
	10	100	101							
	11				110					
	20	200								
	R0 (30)	R00 (300)	R01 (301)	R02 (302)						
	R1 (31)				R10 (310)	R11 (311)				
	R (32)							R0 (320)	R1 (321)	

Identification matrix A is fed into the computer, and the computer sets up the following array.

Allowable Failures in $K_2$ Lots ( $c_2$ )	Number of Lots in $K_2$		
	1	2	3
	0	0	0
	1	1	1
Reject (Failures $> c_2$ )	2	2	2
	$c_2+1$	$c_2+1$	$c_2+1$



From this array, the computer generates the state defining matrix B by testing the sum of every possible combination of the entries in the array against  $c_2$  (one from each column). If the sum of the combination is less than or equal to  $c_2$ , the 3 digits are placed in the appropriate cell of the matrix (B). If the sum exceeds  $c_2$ , the cell is left blank. To illustrate:

Designate the general element of matrix B as (i, j, k) where the permissible values are  $0 \leq i, j, k < (c_2+1)$  and k, j, and i sweep through their permissible values in that order. This yields, for matrix B, ordered triplets of numbers 000, 001, ...,  $00c_2$ ; 010, ...,  $01(c_2-1)$ ; etc.

Elements are placed in sequence after the row is started, and the row is terminated when  $i+j+k > c_2$ . Thus element  $01(c_2-1)$  for the example is 011 and is the last element in row two.

The location of the cell for any element, i j k, may be determined by the pair of two digits numbers (i j) and (j k) designating rows and columns. Element 011 is in row 01 and column 11.

When  $c_2+1$  is reached in column 1 (i in the example), this value is held constant for column i and the restart stage is entered. A count (N) must be kept of the remaining columns in the array, and the sum of defectives in the first  $K_1$  of these columns must be kept  $\leq c_1$ .

With this restriction and  $N > K_1$ , the permissible values of the first  $K_1$  values of the (i j k), following  $i=c_2+1$ , are  $0 \leq j, \dots \leq c_1$ , and the remaining  $N-K_1$  values (k in the example) have permissible values  $0 \leq k \dots \leq c_2$ ,

k and j sweep through their permissible values in that order and with i fixed the triplets of numbers are ordered on j and k. The result is  $c_2+1, j, k = 300, 301, 302; 310, 311$ , where the sum of  $j+k$  is less than or equal to  $c_2$  and the value of the first  $K_1$  columns, j, is less than or equal to  $c_1$ . The triplets are placed in the proper cells as usual; 311 is entered in row 31, column 11, etc.

When  $c_1+1$  is reached in column 2 (j in the example), this value is also held constant and the restart stage continues. Now  $i+j+k$  for the example is  $3+2+k$ , and the permissible values of k are  $0 \leq k \leq c_1$ . The point where  $N=K_1$  has been reached and only restart criteria can apply. The sweep yields triplets ordered on column k of 320 and 321, which are placed in the usual manner into the proper cells. The matrix (B) is now complete. This process is thus generalized to handle any plan automatically.

From the state defining matrix (B), a transition matrix must be established. This is accomplished by taking each ordered triplet and replacing it in the state defining matrix with a binomial probability value. The last digit of an ordered triplet indicates the number of failures occurring in the transition from one state to another. For instance, with a sample size of 10 and a given p, the triplet 002 would be replaced with the binomial probability  $\binom{10}{2} p^2 (1-p)^8$  since in the transition from state 00 to 02 exactly two defects occurred. Replacing all triplets with the corresponding transition probability will generate the transition matrix.

Caution must be exercised when different sample sizes are used for the different stages. The appropriate "n" is used to compute the binomial probability depending upon whether the transition occurred in the restart or normal operating stage.

Once the transition matrix is established, standard computer techniques can be used to form and obtain the steady state solution for the equations defining the state probabilities.

If differing sample sizes are used in the restart and normal operating stage, the computation of the ASN curve can be conveniently done from the same array that was used to generate the state defining matrix (B).

The general expression for ASN computation is

$$\begin{aligned} \text{ASN} &= n_1 P(n_1) + n_2 P(n_2) \\ &= n_1 P(n_1) + n_2 [1 - P(n_1)] \\ &= n_2 + (n_1 - n_2) P(n_1) \end{aligned}$$

where:

- $n_1$  = sample size for restart stage
- $n_2$  = sample size for normal operating stage
- $P(n_1)$  = probability of  $n_1$  being required
- $P(n_2)$  = probability of  $n_2$  being required

In the plan used above to illustrate the generation of the state defining matrix ( $K_1=1$ ,  $c_1=1$ ;  $K_2=3$ ,  $c_2=2$ ), the ASN would simply be:

$$\text{ASN} = n_2 + (n_1 - n_2) (P_R)$$

(where  $P_R = 1 - P_A$ ) since after every rejection a sample of  $n_1$  is required and it is not required for any other condition.

To illustrate the computerization of ASN computation, a more complex example will be chosen. For example, suppose an identification matrix is:

$$\begin{array}{c} K \quad c \quad n \\ 1 \quad \left( \begin{array}{ccc} 3 & 2 & 10 \\ 5 & 3 & 5 \end{array} \right) \\ 2 \end{array}$$

The array used in generating the state defining matrix would be:

	Number of Lots in $K_2$				
	1	2	3	4	5
Allowable Failures in $K_2$ Lots ( $c_2$ )	0	0	0	0	0
	1	1	1	1	1
	2	2	2	2	2
	3	3	3	3	3
Reject ( $\text{Failures} > c_2$ )	$c_2+1$	$c_2+1$	$c_2+1$	$c_2+1$	$c_2+1$

Since after any reject we revert to the restart stage where  $K_1$  is in effect, we need concern ourselves with only the last  $K_1$  columns of the array. In other words, the first  $K_2 - K_1$  columns are ignored. Starting with column 3 set to a reject and looking at the remaining columns for acceptable sequences (based on  $c_1$ ) we have: R00, R01, R02, R10, R11,

and R20. Proceeding and setting column 4 to a reject and generating acceptable sequences, we have: R0, R1, R2. Setting column 5 to reject we generate only R. The sequences we have generated will always be followed by a sample of  $n_1$  and only these sequences can be followed by a sample of  $n_1$ . We have only to compute the probability of each sequence and sum the probabilities to compute  $P(n_1)$ .

$$P(n_1) = P_R P_0 P_0 + P_R P_0 P_1 + P_R P_0 P_2 + P_R P_1 P_0 + P_R P_1 P_1 + P_R P_2 P_0 + P_R P_0 + P_R P_1 + P_R P_2 + P_R$$

$$P(n_1) = P_R + P_R (P_0 P_0 + P_0 P_1 + P_0 P_2 + P_1 P_0 + P_1 P_1 + P_2 P_0 + P_0 + P_1 + P_2)$$

$P_R$  is obtained from the OC curve computation in the steady state solution of the equations defining the steady state probabilities. The  $P_0, P_1, P_2$  terms are binomial probabilities. The computation of ASN can be done from:

$$ASN = n_2 + (n_1 - n_2)P(n_1)$$





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